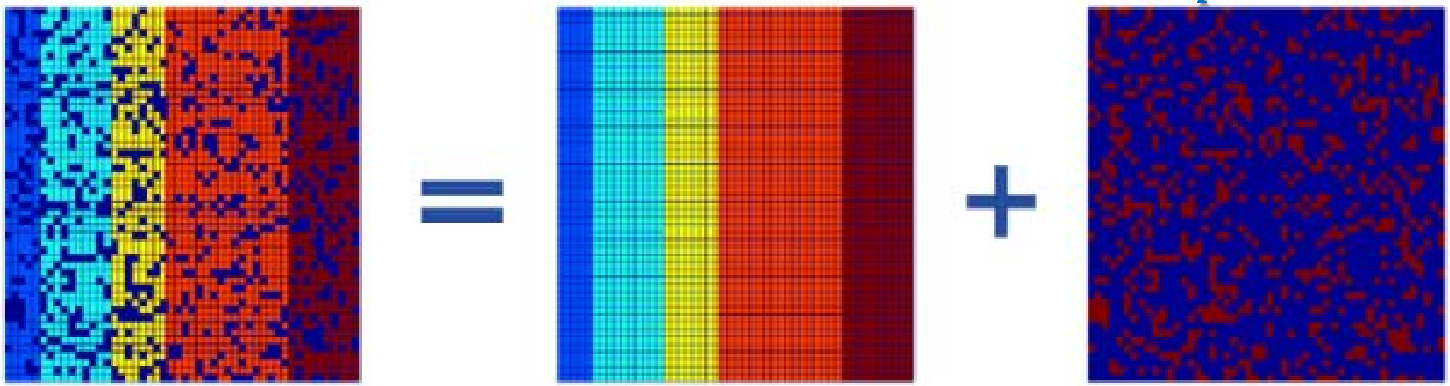
# Efficient Robust Principal Component Analysis via Block Krylov Iteration and CUR Decomposition

**Shun Fang**[1,2], Zhengqin Xu[3], Shiqian Wu[1,2,4*], and Shoulie Xie[5]
[1]*School of Information Science and Engineering, Wuhan University of Science and Technology, China*
[2]*Institute of Robotics and Intelligent Systems, Wuhan University of Science and Technology, China*
[3]*MoE Key Lab of Artificial Intelligence, AI Institute, Shanghai Jiao Tong University, China*
[4]*Hubei Province Key Laboratory of Intelligent Information Processing and Real-Time Industrial Systems*
[5]*Signal Processing, RF & Optical Dept. Institute for Infocomm Research A*STAR, Singapore*
fangshun97@wust.edu.cn, fate311@sjtu.edu.cn, shiqian.wu@wust.edu.cn, slxie@i2r.a-star.edu.sg

# Robust Principal Component Analysis

*observation* *low − rank* *sparse*



$\boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$

- Optimization Model

$$\min_{\boldsymbol{L},\boldsymbol{S}} rank(\boldsymbol{L}) + \lambda\|\boldsymbol{S}\|_0$$
$$s.t.\ \boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$$

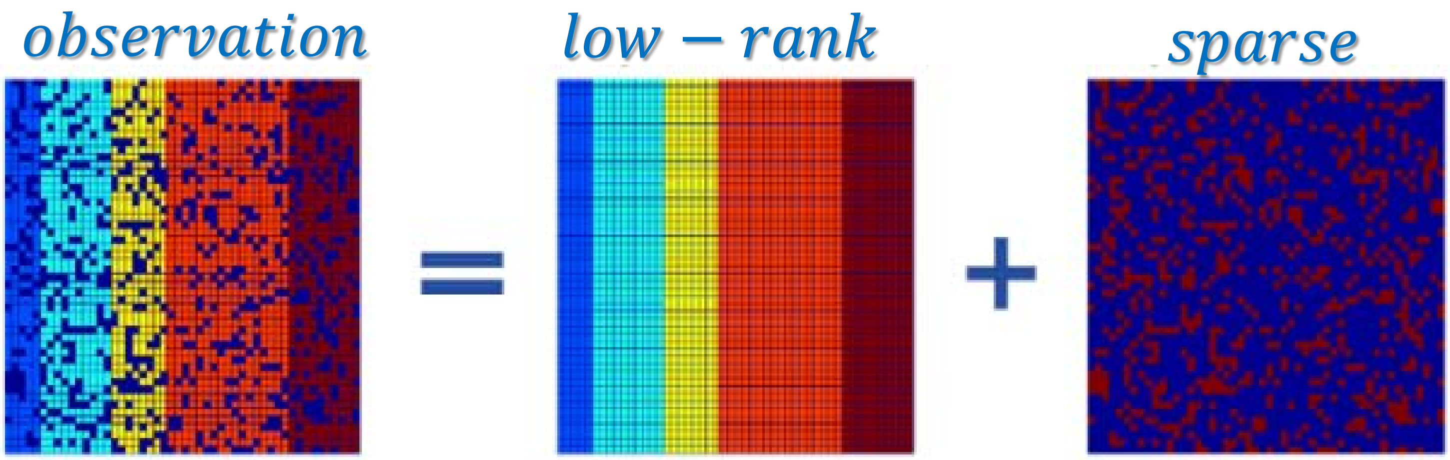
**NP-hard!**

Convex Relaxation ⇒

- Convex Optimization Model

$$\min_{\boldsymbol{L},\boldsymbol{S}} \|\boldsymbol{L}\|_* + \lambda\|\boldsymbol{S}\|_1$$
$$s.t.\ \boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$$

# Robust Principal Component Analysis




$\boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$

- Optimization Model

$$\min_{\boldsymbol{L},\boldsymbol{S}} rank(\boldsymbol{L}) + \lambda\|\boldsymbol{S}\|_0$$
$$s.t.\ \boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$$

**NP-hard!**

Non-Convex Relaxation ⇒

- Non-convex Optimization Model

$$\min_{\boldsymbol{L},\boldsymbol{S}} \|\boldsymbol{M} - \boldsymbol{L} - \boldsymbol{S}\|_F^2$$
$$s.t.\ rank(\boldsymbol{L}) \leq r \text{ and } \|\boldsymbol{S}\|_0 \leq s$$

# Applications of RPCA

Foreground-Background Separation

Shadow Removal from Face Images

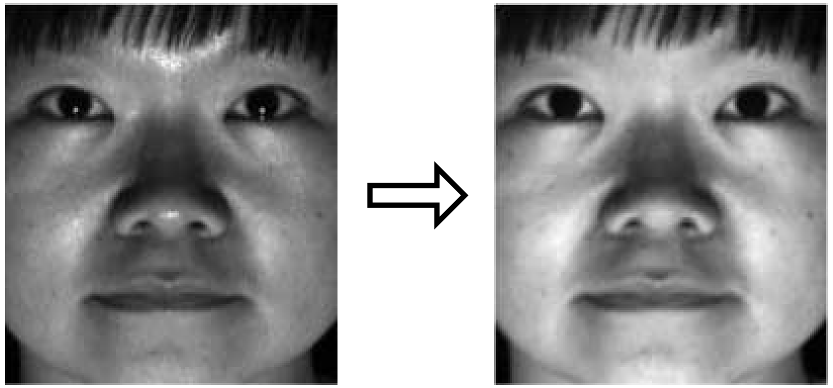
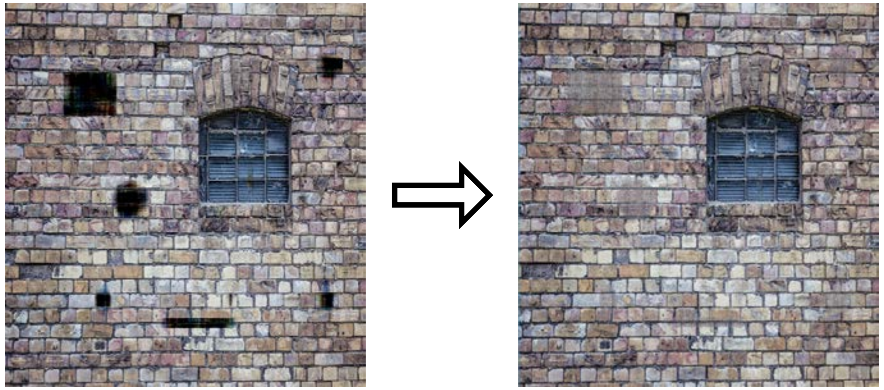
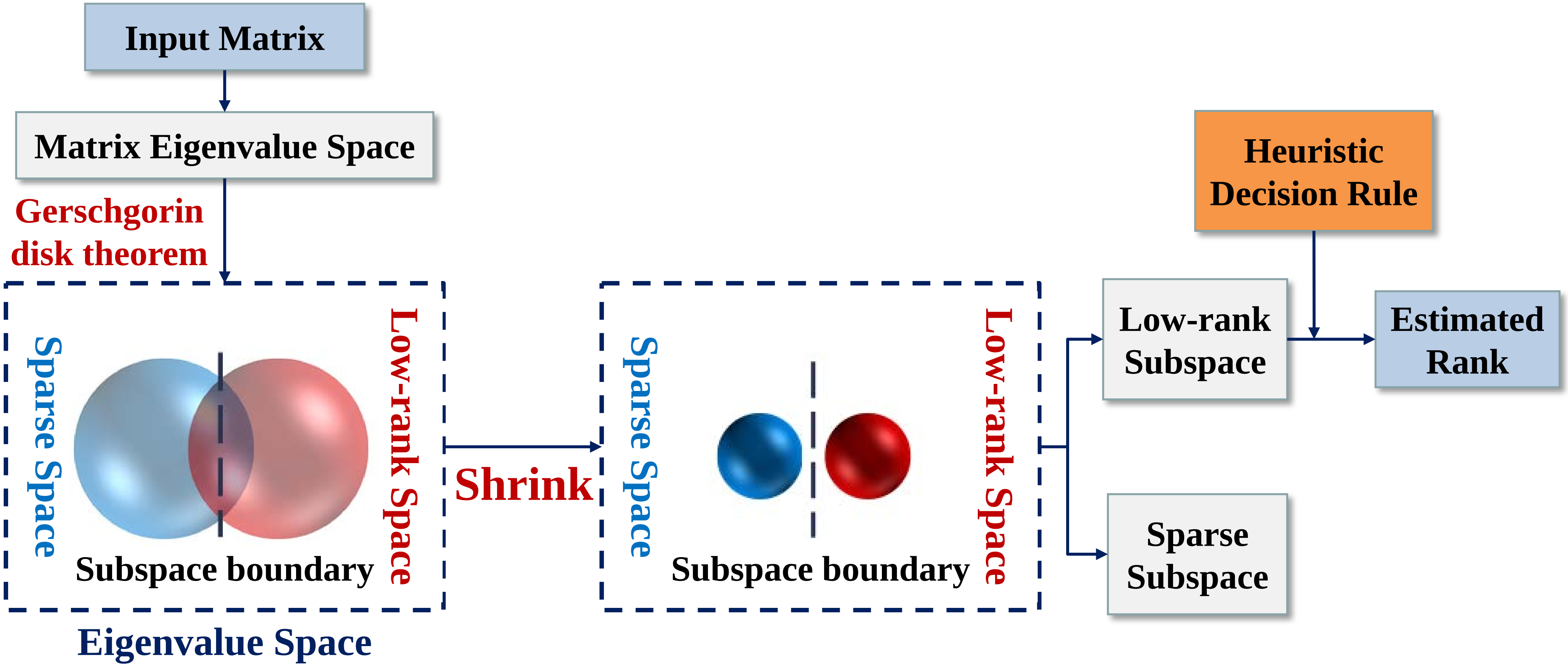


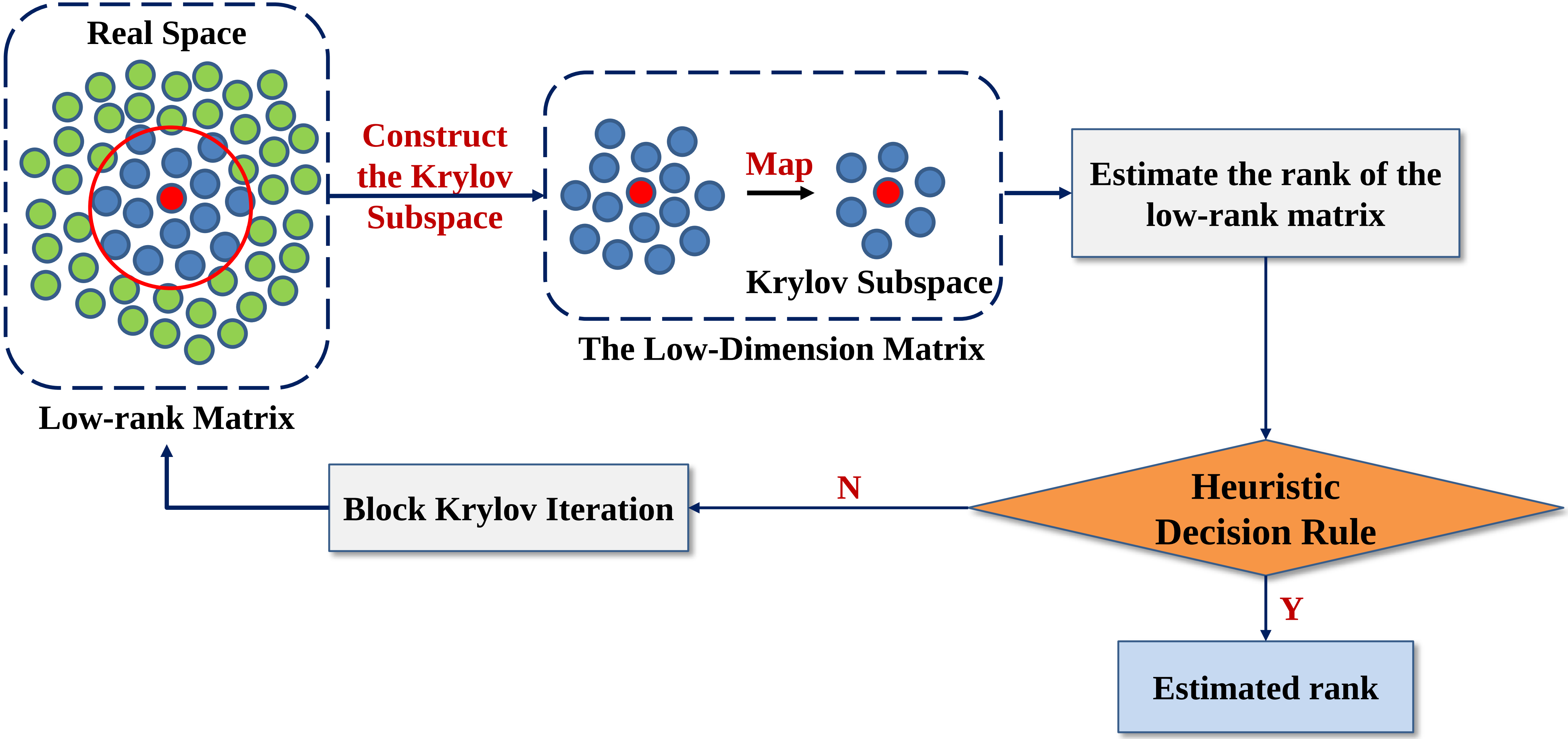
Image Inpainting



# Background


Input Matrix
Matrix Eigenvalue Space
Gerschgorin disk theorem
Sparse Space
Low-rank Space
Subspace boundary
Eigenvalue Space
Shrink
Sparse Space
Low-rank Space
Subspace boundary
Heuristic Decision Rule
Low-rank Subspace
Estimated Rank
Sparse Subspace

# Efficient Rank Estimation



# Efficient RPCA

## ➤ New Optimization Problem

$$\min_{\boldsymbol{L},\boldsymbol{S}} \|\boldsymbol{M}-\boldsymbol{L}-\boldsymbol{S}\|_F^2+\lambda\|\boldsymbol{L}\|_\gamma \quad s.t. \quad \|\boldsymbol{S}\|_0 \le s$$

$$\text{where } \|\boldsymbol{L}\|_\gamma = \begin{cases} \sum_{i=1} \frac{e^\gamma \sigma_i}{\gamma+\sigma_i}, i \le r \\ 0, i > r \end{cases}$$

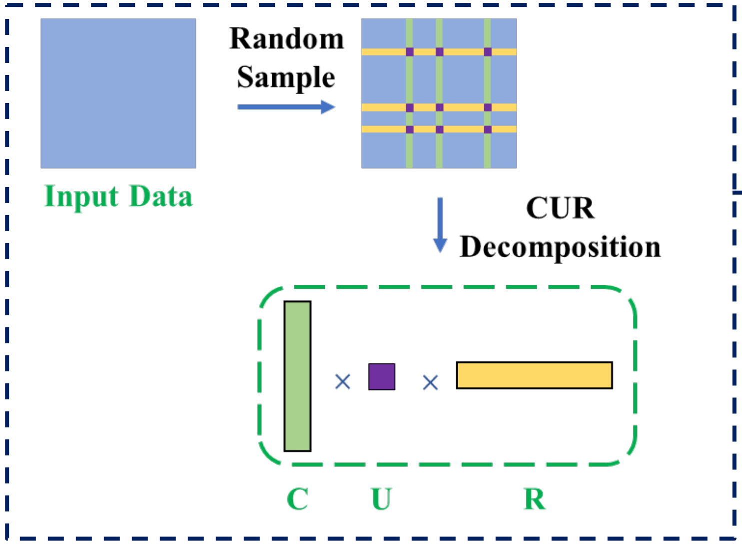
# Efficient RPCA

The rank $r$ of the low-rank matrix

$$\|\boldsymbol{L}\|_{\gamma} = \begin{cases} \sum_{i=1} \frac{e^{\gamma}\sigma_i}{\gamma + \sigma_i}, i \le r \\ 0, i > r \end{cases}$$

$$\min_{\boldsymbol{L},\boldsymbol{S}} \|\boldsymbol{M} - \boldsymbol{L} - \boldsymbol{S}\|_F^2 \quad s.t.\ rank(\boldsymbol{L}) \le r \ and \ \|\boldsymbol{S}\|_0 \le s$$

$$\min_{\boldsymbol{L},\boldsymbol{S}} \|\boldsymbol{M} - \boldsymbol{L} - \boldsymbol{S}\|_F^2 + \lambda \|\boldsymbol{L}\|_{\gamma} \quad s.t.\ \|\boldsymbol{S}\|_0 \le s$$




**CUR Decomposition**

$\boldsymbol{S}$ sub-problems → $\boldsymbol{S}^*$

$\boldsymbol{L}$ sub-problems → $\boldsymbol{L}^*$

# Experimental Results

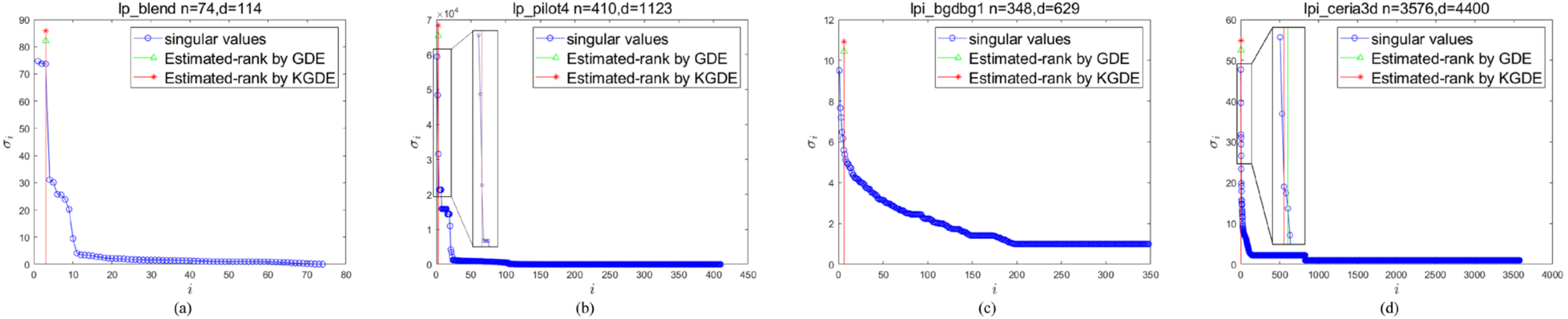



Fig 1. The numerical rank estimation of sparse data matrices.

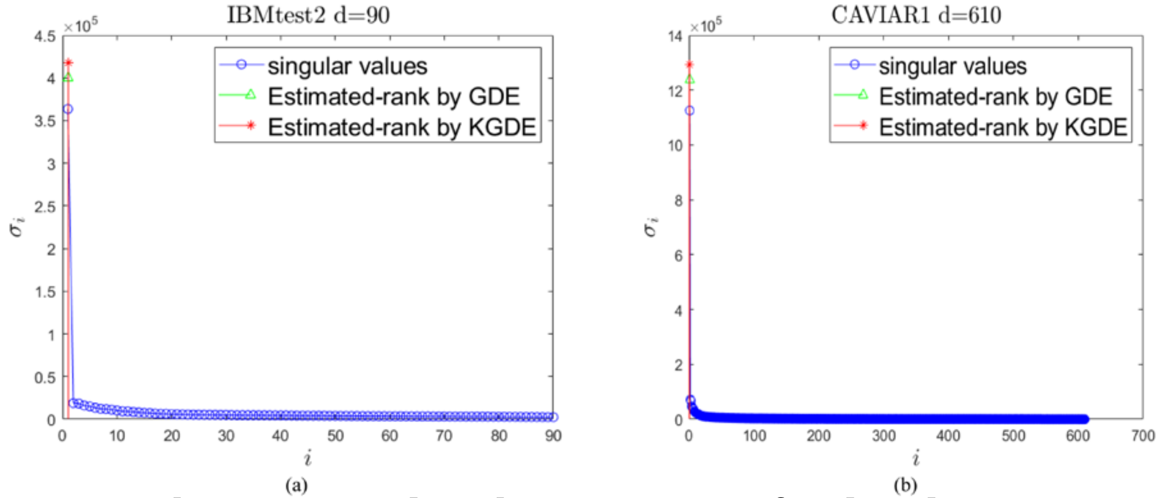



Fig 2. The numerical rank estimation of video data matrices.

Table 1. The results of runtime and estimated rank using GDE and KGDE

| Dataset | GDE Runtime(s) | GDE Rank | KGDE Runtime(s) | KGDE Rank |
|---|---|---|---|---|
| lp_blend | 0.156 | 3 | 0.094 | 3 |
| lp_pilot4 | 1.488 | 3 | 0.250 | 3 |
| lpi_bgdbg1 | 0.274 | 6 | 0.266 | 6 |
| lpi_ceria3d | 91.180 | 5 | 0.234 | 3 |
| IBMtest2 | 0.176 | 1 | 0.134 | 1 |
| CAVIAR1 | 2.235 | 1 | 0.757 | 1 |

# Experimental Results

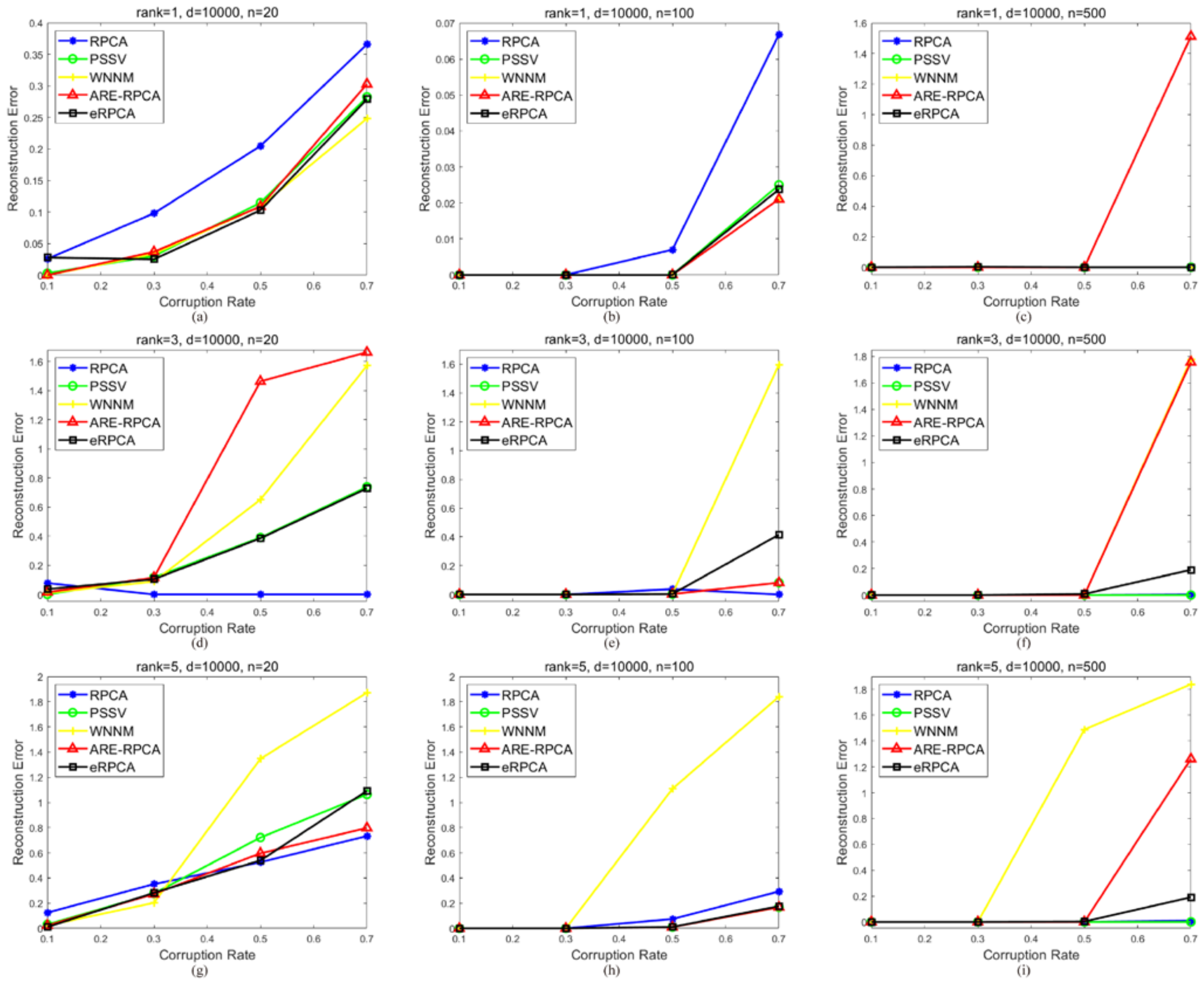



Fig 1. Comparison of various algorithms with different dimensions $n$, matrix ranks $r$, and corruption rates $\alpha$

# Experimental Results

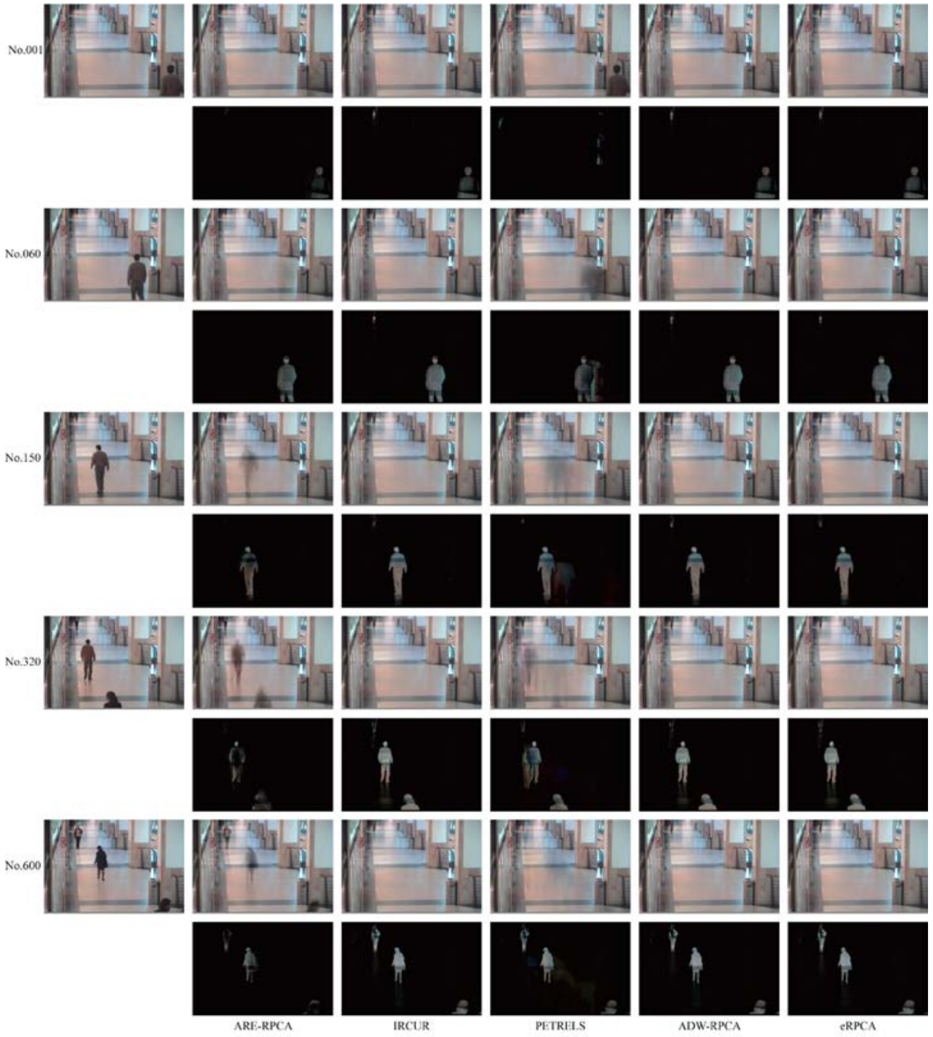



Fig 1. The comparison of visual results on CAVIAR1 between different algorithms.

Table 1. Performance of one frame from CAVIAR1 dataset among different algorithms

| | AGE | pEPs% | pCEPS% | MSSSIM | PSNR | CQM | TIME(s) |
|---|---|---|---|---|---|---|---|
| ARE-RPCA | 5.2637 | 4.0741 | 3.4871 | 0.8564 | 25.4414 | 24.7876 | 10.78593 |
| IRCUR | 3.1489 | 0.2828 | 0.1821 | **0.9917** | 34.4689 | 33.2714 | **0.007564** |
| PETRELS | 4.9972 | 5.0161 | 4.0222 | 0.9329 | 29.8017 | 29.0773 | 1.14708 |
| ADW-RPCA | 2.64 | 0.3499 | 0.2482 | 0.9905 | 34.4853 | 33.3402 | 6.356669 |
| eRPCA | **2.6394** | **0.2686** | **0.1689** | **0.9917** | **34.8626** | **33.5725** | 0.034198 |

# Experimental Results

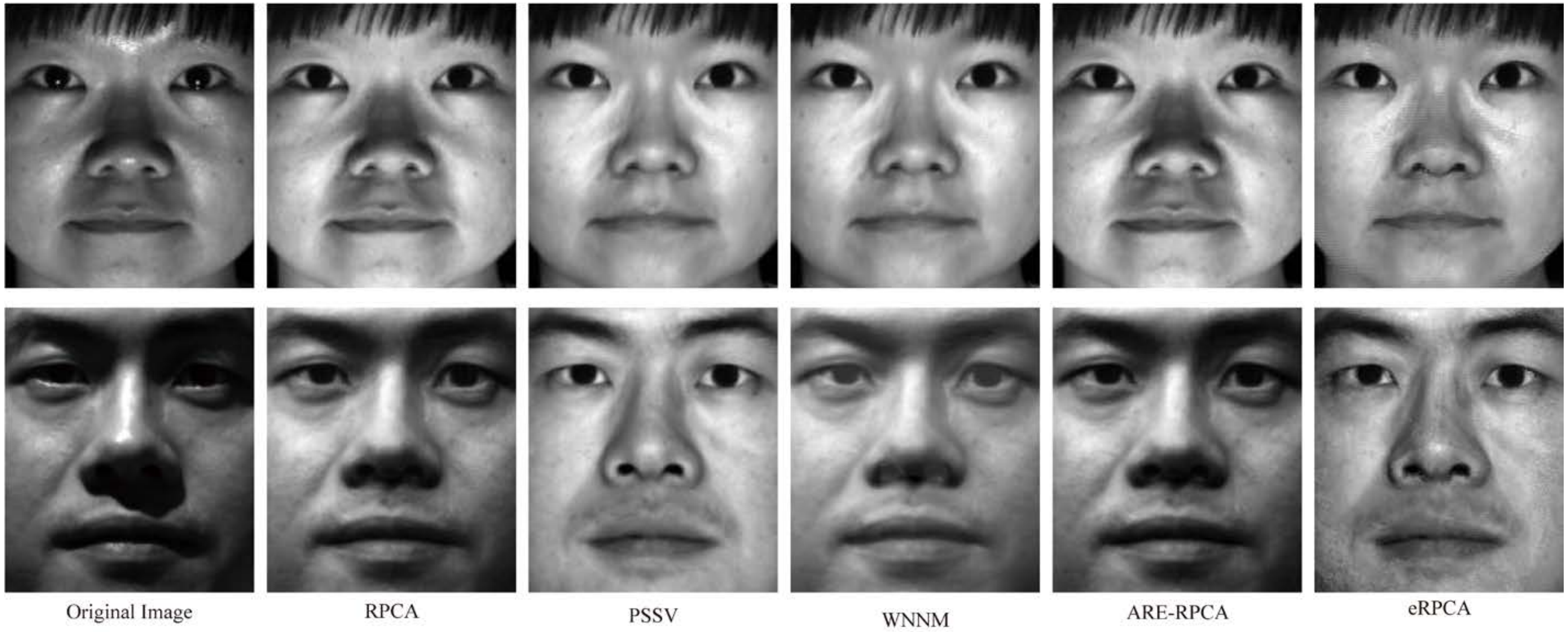



Table 1. The comparison of runtime between different algorithms

| | RPCA | PSSV | WNNM | ARE-RPCA | eRPCA |
|---|---|---|---|---|---|
| yale05 | 3.535s | 0.973s | 1.113s | 3.648s | **0.071s** |
| yaleB02 | 3.499s | 0.962s | 2.424s | 3.650s | **0.081s** |

# Conclusion

## Advantage

- High accuracy
- Efficient
- Easy implementation