Rethinking the Approximation Error in 3D Surface Fitting for Point Cloud Normal Estimation

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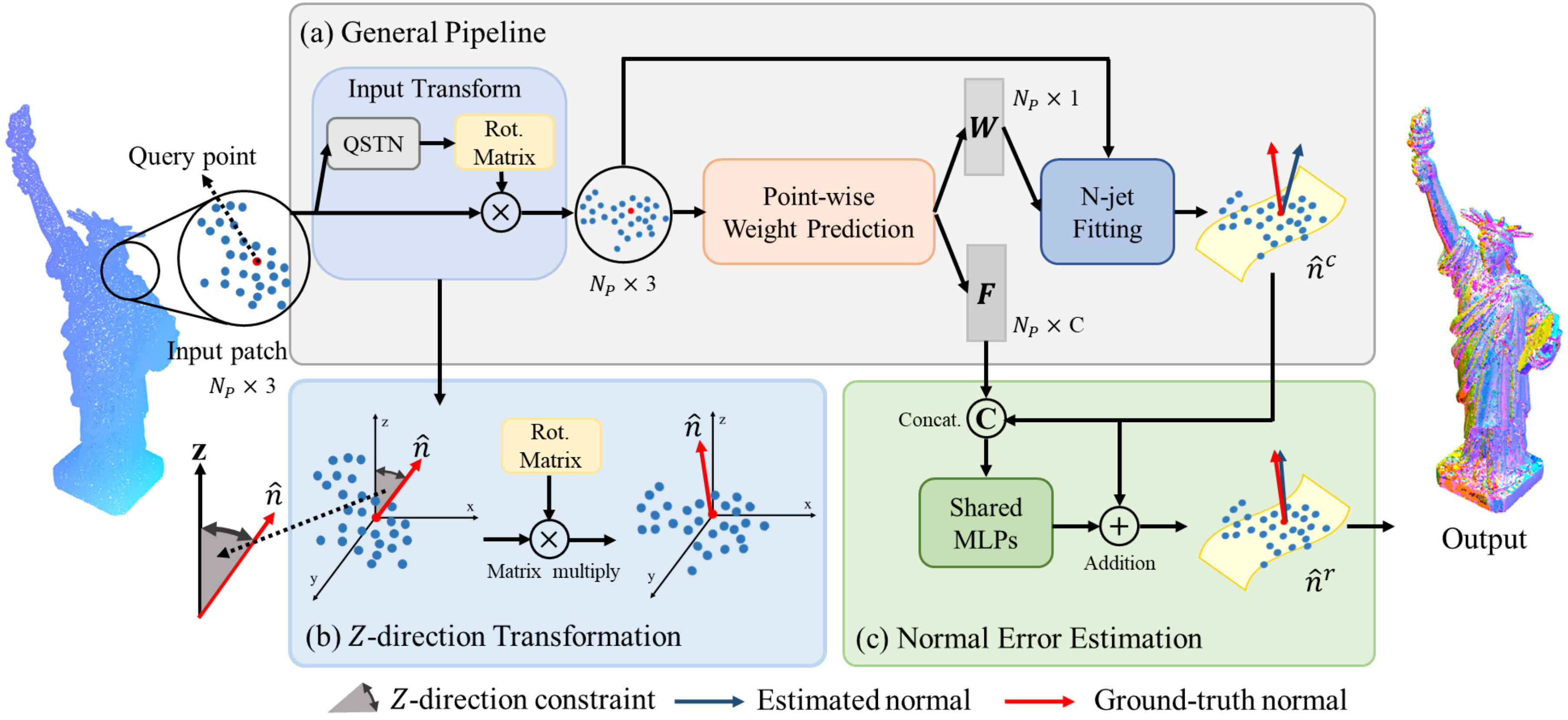
Quick preview

To bridge the gap between estimated and precise surface normals, two basic deign principles are proposed.

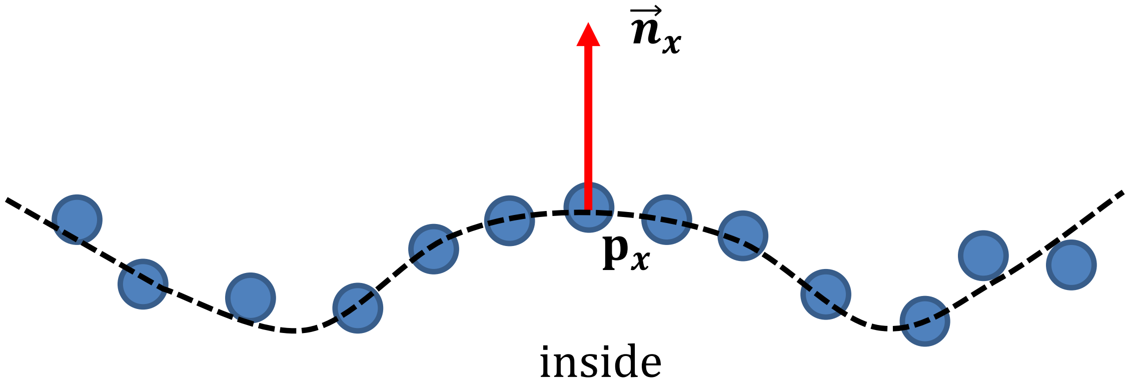
- applies a z-alignment transformation for a better surface fitting.
- models the error of normal estimation as a learnable term to compensate the rough estimated normal.



Problem formulation

Given a point cloud patch $\boldsymbol{\mathcal{P}} = \{\mathbf{p}_{i} \in \mathbb{R}^{3} \mid (p_i^x, p_i^y, p_i^z)\}_{\mathbf{i=1}}^{N_p}$, predict a normal vector $\vec{\boldsymbol{n}}_x$ at a query point $\mathbf{p}_x$.

1) Direction: the local region of the query point.
2) Orientation: globally consistent, outside or inside of the surface.



Problem formulation

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Revisiting n-jet surface fitting theory:

An order n Taylor expansion of the height function over a surface is defined as

$$f(x,y) = J_{\beta,n}(x,y) + O(\| (x,y) \|^{n+1}).$$

Then, the normal vector can be calculated by

$$\vec{\boldsymbol{n}} = \frac{(-\beta_{1,0}, -\beta_{0,1}, 1)}{\sqrt{\beta_{1,0}^2 + \beta_{0,1}^2 + 1}}.$$

The n-jet surface model aims to find an approximation result on the coefficients of the height function,

$$J_{\alpha,n} = argmin\left\{\sum_{i=1}^{N_p} \left(J_{\alpha,n}(x_i,y_i) - f(x_i,y_i)\right)^2\right\}.$$

Analysis

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Assuming the convergence rate of approximation is given by $O(h) = \| (x, y) \|$, the coefficients $\beta_{k-j,j}$ of $J_{\beta,n}(x, y)$ are estimated by those of $J_{\alpha,n}(x, y)$ up to accuracy $O\left(h^{n-k+1}\right)$:

$$\alpha_{k-j,j} = \beta_{k-j,j} + \mathbf{O}\left(\boldsymbol{h}^{\boldsymbol{n-k+1}}\right)$$

A smaller error $\mathbf{O}\left(\boldsymbol{h}^{\boldsymbol{n-k+1}}\right)$ enables to yield a more precise estimation.

Analysis

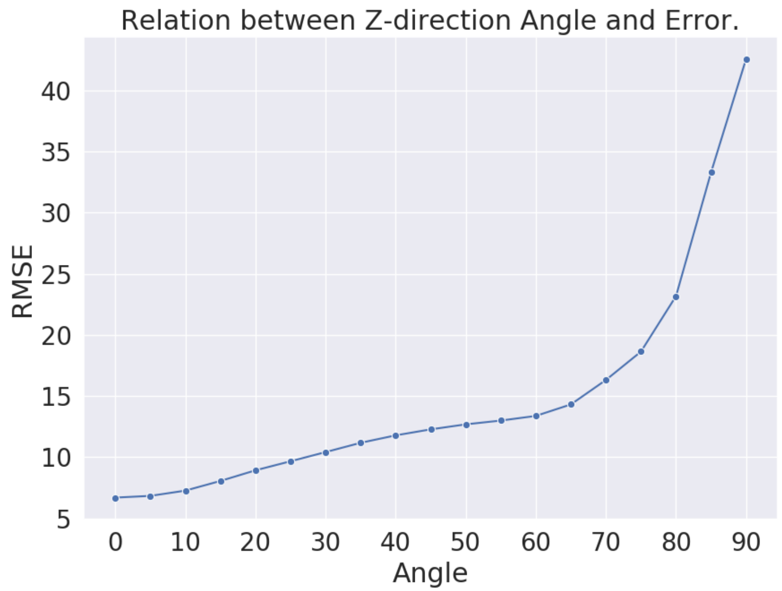
1) A better z-alignment leads to a more accurate fitting: *the error estimates are better when the convex hull K of the sample points* $\{(xi, yi)\}_{i=1,\dots,N}$ *is not too "flat".*

$$O\left(h^{n-k+1}\right) \leq \sup\left\{\| D^k f(x,y) - D^k J_{\alpha,n}(x,y) \|; (x,y) \in K\right\}$$

$O\left(h^{n-k+1}\right)$ *depends on the supremum of* $\{\| D^{n+1} f(x,y) \|; (x,y) \in K\}$

$d_{max} = diameter\ (K)\ ,\ d_{min} = \sup\ \{diameter\ of\ disks\ inscribed\ in\ K\}$

A small ratio d_{max}/d_{min} *means that the geometry of K is not too "flat"*



Relation between z-direction angle and estimation error

Analysis

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2) The presence of error term in normal estimation: *a polynomial fitting of degree n estimates any kth-order differential quantity to accuracy* $O(h^{n-k+1})$.

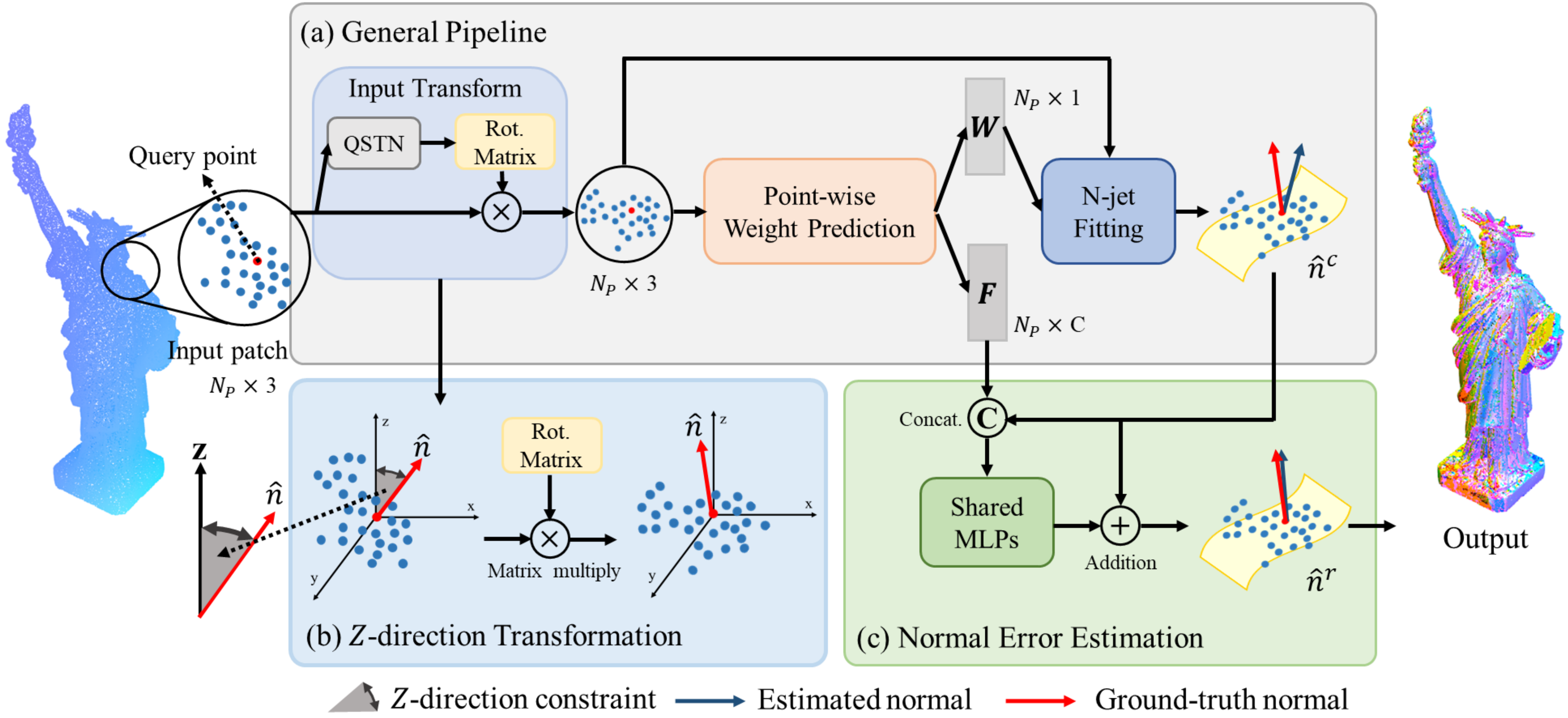
Taking a curve for instance:

$$F(\alpha_{i=0,\dots,k}) = F\left(\beta_{i=0,\dots,k} + O(h^{n-k+1})\right)$$
$$= F(\beta_{i=0,\dots,k}) + DF_{\left(\beta_i + O(h^{n-k+1})\right)_{i=0,\dots,k}}\left(O(h^{n-k+1})\right).$$

For normal estimation, *i.e.*, $k = 1$, the error term $DF(O(h^n))$ denotes the angle between the true normal and the estimated normal.

The proposed approach

Overview architecture.



The proposed approach

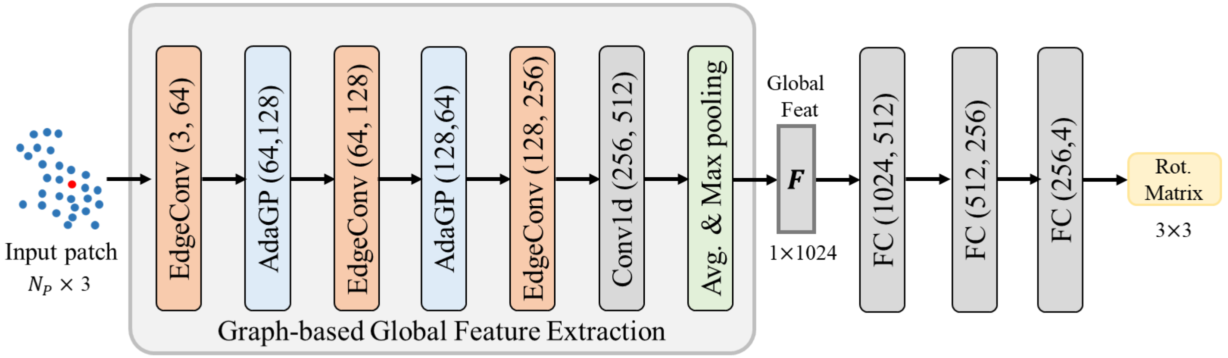
1) Z-direction Transformation: explicitly learns a z-alignment transformation $\mathbf{T}$ that rotates input patches for a better surface fitting.

Training supervision:

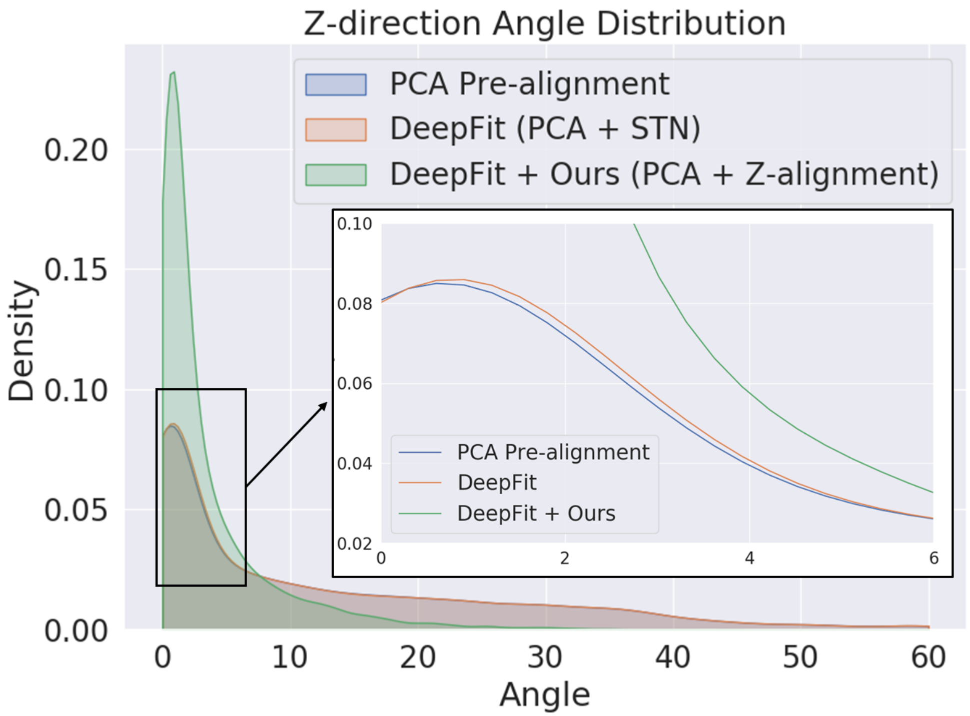
$$\hat{\mathbf{z}} = (0,0,1)^{\top}$$

$$\mathcal{L}_{\text{trans}} = |\hat{\mathbf{n}}_i \mathbf{T} \times \hat{\mathbf{z}}|$$

GCN-based spatial transformation network:



The distribution of z-direction angle:



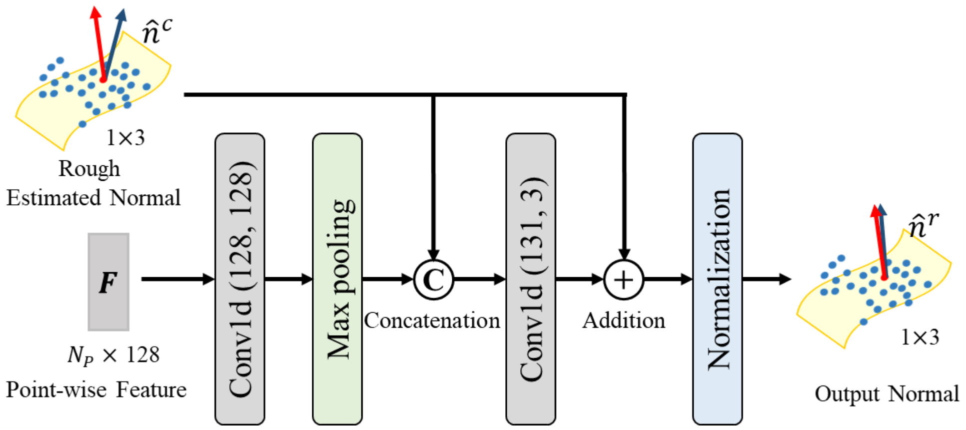
The proposed approach

2) Normal Error Estimation: models the error of normal estimation $DF\big(O(h^n)\big)$ as a learnable term which compensates the rough estimated surface normal.

$$DF\big(O(h^n)\big) = \Delta(\hat{\mathbf{n}}_i^c) = \phi(\mathrm{Concat}(x_i, \hat{\mathbf{n}}_i^c)),$$

$$\hat{\mathbf{n}}_i^r = \hat{\mathbf{n}}_i^c + \Delta(\hat{\mathbf{n}}_i^c).$$

The normal error estimation network.



Experimental results

Normal angle RMSE of our methods and baseline models on PCPNet dataset.

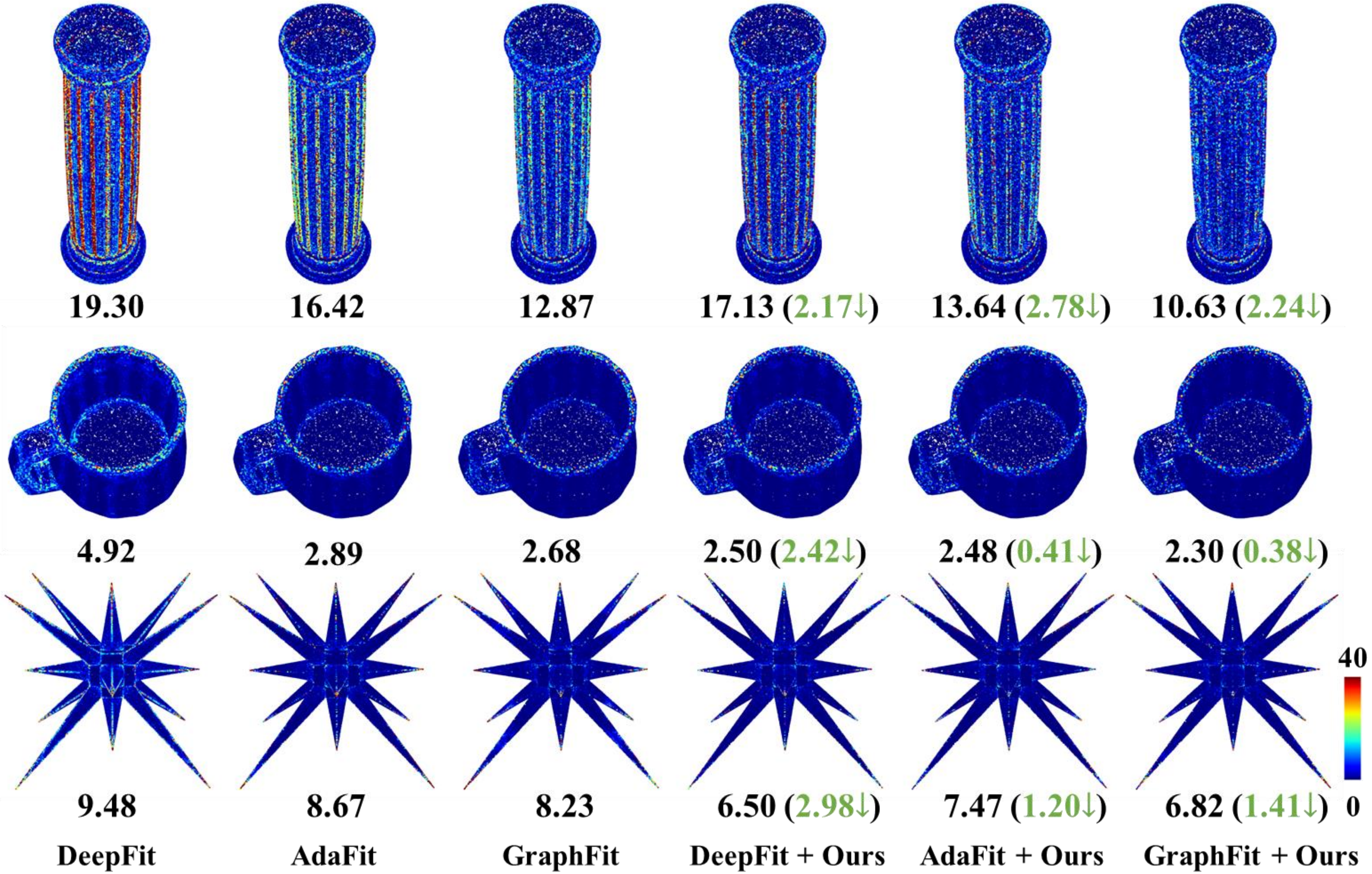
Aug.	GrapFit + Ours	AdaFit + Ours	DeepFit + Ours	GraphFit [21]	AdaFit [42]	DeepFit [3]	IterNet [19]	Nesti-Net [5]	PCPNet [13]	Jet [8]	PCA [15]
No Noise	**4.11**	4.71	4.90	4.45	5.19	6.51	6.72	6.99	9.62	12.25	12.29
Noise ($\sigma = 0.125\%$)	**8.66**	8.75	8.91	8.74	9.05	9.21	9.95	10.11	11.37	12.84	12.87
Noise ($\sigma = 0.6\%$)	**16.02**	16.31	16.61	16.05	16.44	16.72	17.18	17.63	18.87	18.33	18.38
Noise ($\sigma = 1.2\%$)	**21.57**	21.64	22.87	21.64	21.94	23.12	21.96	22.28	23.28	27.68	27.50
Density (Gradient)	**4.83**	5.51	5.52	5.22	5.90	7.31	7.73	9.00	11.70	13.13	12.81
Density (Striped)	**4.89**	5.48	5.70	5.48	6.01	7.92	7.51	8.47	11.16	13.39	13.66
Average	**10.01** (**0.25↓**)	10.40 (**0.36↓**)	10.75 (**1.05↓**)	10.26	10.76	11.80	11.84	12.41	14.34	16.29	16.25

Comparison of percentage of good points PGP5 and PGP10 on PCPNet dataset.

Method	DeepFit		AdaFit		GraphFit	
	PGP5	PGP10	PGP5	PGP10	PGP5	PGP10
Baseline	80.03	90.72	88.24	94.36	89.73	95.66
+ Ours	89.83	95.59	90.40	95.82	**91.28**	**96.59**

Experimental results

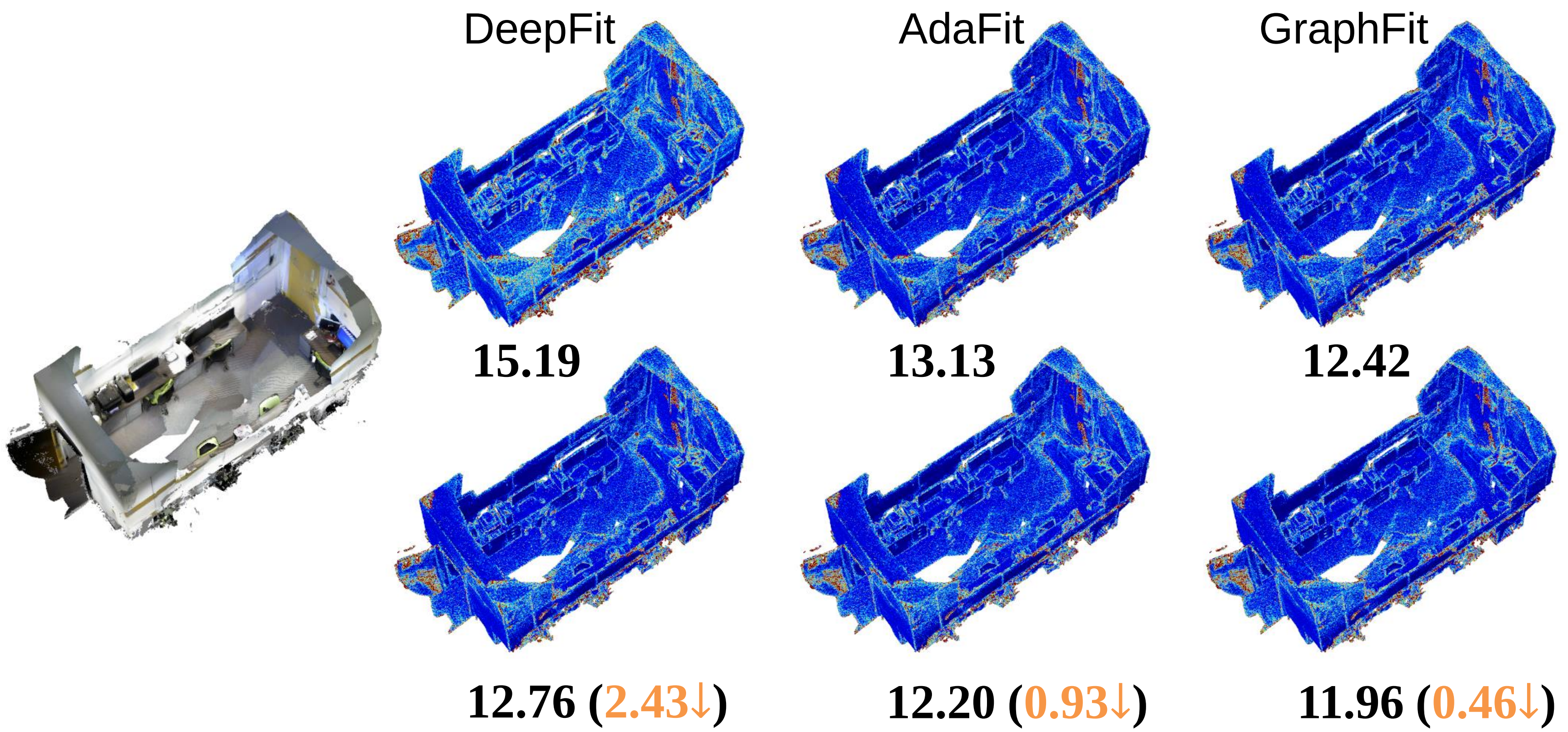
Qualitative results on PCPNet dataset.



Experimental results

Normal angle RMSE SceneNN real-world dataset.

Method	DeepFit	AdaFit	GraphFit
Baseline	17.13	15.49	14.79
+ Ours	14.57 (**2.56↓**)	14.45 (**1.04↓**)	14.51 (**0.28↓**)



Ablation study

Normal angle RMSE of SOTA models with or without our methods on PCPNet dataset.

Aug.	DeepFit (size = 256)				AdaFit (size = 700)				GraphFit (size = 500)			
Z-direction Trans.		✓		✓		✓		✓		✓		✓
Error Estimation			✓	✓			✓	✓			✓	✓
No Noise	6.51	6.27	5.01	4.90	5.19	4.93	4.72	4.71	4.45	4.27	4.36	4.11
Low Noise	9.21	9.10	9.09	8.91	9.05	8.94	8.81	8.75	8.74	8.79	8.71	8.66
Med Noise	16.72	16.68	16.66	16.61	16.44	16.39	16.34	16.31	16.05	16.02	16.05	16.02
High Noise	23.12	22.98	22.94	22.87	21.94	21.56	21.80	21.64	21.64	21.66	21.60	21.57
Gradient	7.31	7.17	5.69	5.52	5.90	5.63	5.59	5.51	5.22	4.98	5.06	4.83
Striped	7.92	7.73	5.85	5.70	6.01	5.89	5.62	5.48	5.48	5.10	5.18	4.89
Average	11.80	11.66	10.87	**10.75**	10.76	10.56	10.48	**10.40**	10.26	10.14	10.16	**10.01**

Summary

The main contributions of this study:

1) analysis of the approximation error in n-jet surface fitting.
2) two basic design principles to improve the precision of point cloud normal estimation.
3) flexible integration with the current polynomial surface fitting methods.
4) extensive experiments to show the improvements by the our method.

Summary

Thanks for your watching!

Models and code are publicly available:
https://github.com/hikvision-research/3DVision