



# Regularizing Second-Order Influences for Continual Learning

Zhicheng Sun<sup>1</sup>, Yadong Mu<sup>1,2</sup>\*, Gang Hua<sup>3</sup>

<sup>1</sup>Peking University, <sup>2</sup>Peng Cheng Laboratory, <sup>3</sup>Wormpex AI Research

### Quick Preview

- **Continual learning** aims to learn on long task sequences without catastrophic forgetting.
- **Replay-based methods** address this by rehearsing on a small replay buffer, which requires careful sample selection.
- However, existing strategies are designed for single-round selection, neglecting the interactions between selection steps.
- This work proposes to model the interactions with **influence functions** and address it via a regularized selection strategy.





# Introduction

Task description

- Continual learning<sup>[1]</sup> studies the training of models on long task sequences with potential data distribution shift.
- It is known for suffering from catastrophic forgetting<sup>[2]</sup>, where the model abruptly forgets past knowledge after being updated on new tasks.





Zhiyuan Chen and Bing Liu. Lifelong Machine Learning. Morgan & Claypool Publishers, 2018.
Michael McCloskey and Neal J Cohen. "Catastrophic Interference in Connectionist Networks: The Sequential Learning Problem". Psychology of Learning and Motivation, 1989, 24: 109–165.

# Introduction

Motivation

 Replay-based approaches mitigate forgetting by rehearsing on a small replay buffer, which requires careful sample selection.

 However, existing selection strategies primarily focus on refining single-round performance, neglecting the interactions between consecutive selection steps through the data flow.



# Introduction

Our contributions

• We investigate the interaction between consecutive selection steps in continual learning and identify a new class of second-order influences.

 A novel regularizer is proposed to mitigate secondorder influences, which also has clear connection to two other popular selection criteria.



Problem formulation

• We consider learning on a data stream  $Z_{1:t} = \bigcup_{i=1}^{t} Z_i$  with a small coreset  $C_t$ . The sample selection goal is to preserve performance on  $C_{t-1} \cup Z_t$  by replaying on  $C_t$ :

$$\min_{C_t \subset C_{t-1} \cup \mathcal{Z}_t, |C_t| \le m} \sum_{\substack{z_i \in C_{t-1} \cup \mathcal{Z}_t}} L(z_i, \hat{\theta})$$
  
s.t.  $\hat{\theta} = \arg\min_{\theta} \sum_{z_i \in C_t} L(z_i, \theta).$ 

 In the following, we will first present a greedy solution based on influence functions<sup>[1,2]</sup>, then showcase its limitations and propose our improved version.

Frank R Hampel. "The Influence Curve and Its Role in Robust Estimation". Journal of the American Statistical Association, 1974, 69(346): 383–393.
Pang Wei Koh and Percy Liang. "Understanding Black-Box Predictions via Influence Functions". In: ICML. 2017: 1885–1894.

Influence-based selection

• To solve the bilevel optimization problem, we linearly approximate the effect of selecting each sample *z* by perturbing its weight:

$$\hat{\theta}_{\epsilon,z} = \arg\min_{\theta} \sum_{z_i \in C_t} L(z_i, \theta) + \epsilon L(z, \theta).$$

• A classic result<sup>[1]</sup> gives the influence of upweighting z on the outer loss:

$$I(z) = \sum_{z_i \in C_{t-1} \cup \mathcal{Z}_t} \frac{dL(z_i, \hat{\theta}_{\epsilon, z})}{d\epsilon} \Big|_{\epsilon=0}$$
  
=  $-\sum_{z_i \in C_{t-1} \cup \mathcal{Z}_t} \nabla_{\theta} L(z_i, \hat{\theta}_t)^{\mathsf{T}} H_{\hat{\theta}_t}^{-1} \nabla_{\theta} L(z, \hat{\theta}_t).$ 

• It further yields an optimal solution that greedily select the most influential samples.

[1] R Dennis Cook and Sanford Weisberg. Residuals and Influence in Regression. New York: Chapman and Hall, 1982.

Second-order influences

• This greedy selection strategy favors samples that are more similar to the existing ones.

$$\mathcal{I}(z) = -\sum_{z_i \in \mathcal{C}_{t-1} \cup \mathcal{Z}_t} \nabla_{\theta} L(z_i, \hat{\theta}_t)^\top H_{\hat{\theta}_t}^{-1} \nabla_{\theta} L(z, \hat{\theta}_t).$$

• Due to second-order effects, it would result in a biased and less diversified coreset:



Second-order influences

- To model such an effect, we upweight two samples z and z' from consecutive selection steps. Upweighting the previous sample interferes with the subsequent selection:
- If z and z' are not jointly optimized in the next round:

$$\mathcal{I}_{\epsilon,z}(z') = -\left(\sum_{z_i \in C_t \cup \mathcal{Z}_{t+1}} \nabla_{\theta} L(z_i, \hat{\theta}_{t+1}) + \epsilon \nabla_{\theta} L(z, \hat{\theta}_{t+1})\right)^{\mathsf{T}} H_{\hat{\theta}_{t+1}}^{-1} \nabla_{\theta} L(z', \hat{\theta}_{t+1}).$$
$$\mathcal{I}^{(2)}(z, z') = -\nabla_{\theta} L(z, \hat{\theta}_{t+1})^{\mathsf{T}} H_{\hat{\theta}_{t+1}}^{-1} \nabla_{\theta} L(z', \hat{\theta}_{t+1}).$$

• If z and z' are jointly optimized in the next round:

$$I_{\epsilon,z}(z') = -\left(\sum_{z_i \in C_t \cup Z_{t+1}} \nabla_{\theta} L(z_i, \hat{\theta}_{t+1}) + \epsilon \nabla_{\theta} L(z, \hat{\theta}_{t+1})\right)^{\mathsf{T}} (H_{\hat{\theta}_{t+1}} + \epsilon H_{\hat{\theta}_{t+1},z})^{-1} \nabla_{\theta} L(z', \hat{\theta}_{t+1}).$$
$$I^{(2)}(z, z') = -(\nabla_{\theta} L(z, \hat{\theta}_{t+1}) - H_{\hat{\theta}_{t+1},z} s_{t+1})^{\mathsf{T}} H_{\hat{\theta}_{t+1}}^{-1} \nabla_{\theta} L(z', \hat{\theta}_{t+1}).$$

Regularizing influences

• The total interference is a weighted sum of the two second-order influences:

$$\begin{split} \Delta I(z') &\approx -\sum_{z \in \overline{C}_t} I^{(2)}(z,z') \cdot 1 \\ &= \sum_{z \in \overline{C}_t} (\nabla_{\theta} L(z,\hat{\theta}_{t+1}) - \mu H_{\hat{\theta}_{t+1},z} s_{t+1})^T H_{\hat{\theta}_{t+1}}^{-1} \nabla_{\theta} L(z',\hat{\theta}_{t+1}). \end{split}$$

• Its magnitude can be upper-bounded with the following regularizer:

$$\begin{split} |\Delta I(z')| &\leq \left\| \sum_{z \in \overline{C}_t} \left( \nabla_{\theta} L(z, \hat{\theta}_{t+1}) - \mu H_{\hat{\theta}_{t+1}, z} s_{t+1} \right) \right\| \times \left\| H_{\hat{\theta}_{t+1}}^{-1} \nabla_{\theta} L(z', \hat{\theta}_{t+1}) \right\|, \\ \mathcal{R}(C_t) &= \left\| \sum_{z \in \overline{C}_t} \left( \nabla_{\theta} L(z, \hat{\theta}_t) - \mu H_{\hat{\theta}_t, z} s_t \right) \right\| \end{split}$$

• This regularizer is used in the final selection criterion: minimize  $\sum_{z \in C_t} I(z) + v \mathcal{R}(C_t)$ .

Interpreting the regularizer

$$\mathcal{R}(C_t) = \left\| \sum_{z \in \overline{C}_t} (\nabla_{\theta} L(z, \hat{\theta}_t) - \mu H_{\hat{\theta}_t, z} s_t) \right\|$$

- $\mu = 0 \Rightarrow \text{gradient matching}^{[1]}$ :  $\mathcal{R}(C_t) = \left\| \sum_{z \in C_{t-1} \cup Z_t} \nabla_{\theta} L(z, \hat{\theta}_t) - \sum_{z \in C_t} \nabla_{\theta} L(z, \hat{\theta}_t) \right\|.$
- $\mu > 0$ , identical Hessian  $\Rightarrow$  diversity<sup>[2]</sup>:

$$\mathcal{R}(C_t) = \left\| (1 - \alpha \mu) \sum_{z \in C_{t-1} \cup \mathcal{Z}_t} \nabla_{\theta} L(z, \hat{\theta}_t) - \sum_{z \in C_t} \nabla_{\theta} L(z, \hat{\theta}_t) \right\|,$$

• additional Hessian-related information



# Experiments

• Comparison to state-of-the-art methods on Split CIFAR-10

Method		Class-incremental				Task-incremental				
		m = 300		m = 500		m = 300		m = 500		
		ACC (%)	BWT (%)	ACC (%)	BWT (%)	ACC (%)	BWT (%)	ACC (%)	BWT (%)	
Non-IF	GEM <sup>[12]</sup>	37.51	-70.48	36.95	-69.76	89.34	-9.09	90.42	-7.88	
	A-GEM <sup>[93]</sup>	20.02	-95.68	20.01	-95.69	85.52	-14.07	86.45	-12.83	
	ER <sup>[89]</sup>	34.19	-78.18	40.45	-70.36	88.97	-9.95	90.60	-7.74	
	GSS <sup>[22]</sup>	35.89	-75.80	41.96	-68.24	88.05	-10.63	90.38	-7.73	
	ER-MIR <sup>[94]</sup>	38.53	-72.72	42.65	-67.50	88.50	-10.33	90.63	-7.62	
	GDUMB <sup>[21]</sup>	36.92	-	44.27	-	73.22	-	78.06	-	
	HAL <sup>[95]</sup>	24.45	-83.56	27.94	-80.01	79.90	-14.39	81.84	-12.73	
	GMED <sup>[96]</sup>	38.12	-73.16	43.68	-66.21	88.91	-9.76	89.72	-8.75	
IF	Vanilla IF	41.76	-68.59	47.14	-62.20	90.67	-7.65	91.06	-7.36	
	MetaSP <sup>[36]</sup>	43.76	-66.37	50.10	-58.39	89.91	-9.00	91.41	-7.36	
	Ours	48.62	-60.24	53.07	-54.44	91.52	-6.94	92.53	-5.46	

# Experiments

• Comparison to state-of-the-art methods on Split *mini*lmageNet

Method		Class-incremental				Task-incremental				
		m = 500		m = 1000		m = 500		m = 1000		
		ACC (%)	BWT (%)	ACC (%)	BWT (%)	ACC (%)	BWT (%)	ACC (%)	BWT (%)	
Non-IF	A-GEM <sup>[93]</sup>	10.69	-49.22	10.69	-49.16	18.34	-39.65	18.78	-39.05	
	ER <sup>[89]</sup>	11.00	-50.84	11.35	-50.08	28.97	-28.40	31.59	-24.95	
	GSS <sup>[22]</sup>	11.09	-50.66	11.42	-49.91	28.67	-28.71	31.75	-24.56	
	ER-MIR <sup>[94]</sup>	11.07	-50.46	11.32	-49.92	29.10	-27.95	31.39	-24.89	
	GDUMB <sup>[21]</sup>	6.22	-	7.15	-	16.37	-	17.69	-	
	GMED <sup>[96]</sup>	11.03	-50.23	11.73	-48.93	30.47	-26.02	32.85	-22.69	
IF	Vanilla IF	12.08	-48.55	14.64	-47.15	33.74	-21.71	37.55	-19.28	
	MetaSP <sup>[36]</sup>	12.74	-48.84	14.54	-45.52	34.36	-21.70	37.20	-17.83	
	Ours	13.63	-47.94	16.15	-43.78	36.46	-19.48	39.61	-16.01	

### Experiments

• Ablation studies of hyperparameter sensitivity and influence estimation accuracy



### Thanks for listening

Code is available at <a href="https://github.com/feifeiobama/InfluenceCL">https://github.com/feifeiobama/InfluenceCL</a>