

## Motivation

- Training cost scales with number of images and acquiring new training views is expensive.
- Current information formulations for 3D-GS are rigid/constrained to one function.
- Classical approaches formulate view selection through optimal experimental design, yet application is non-trivial.

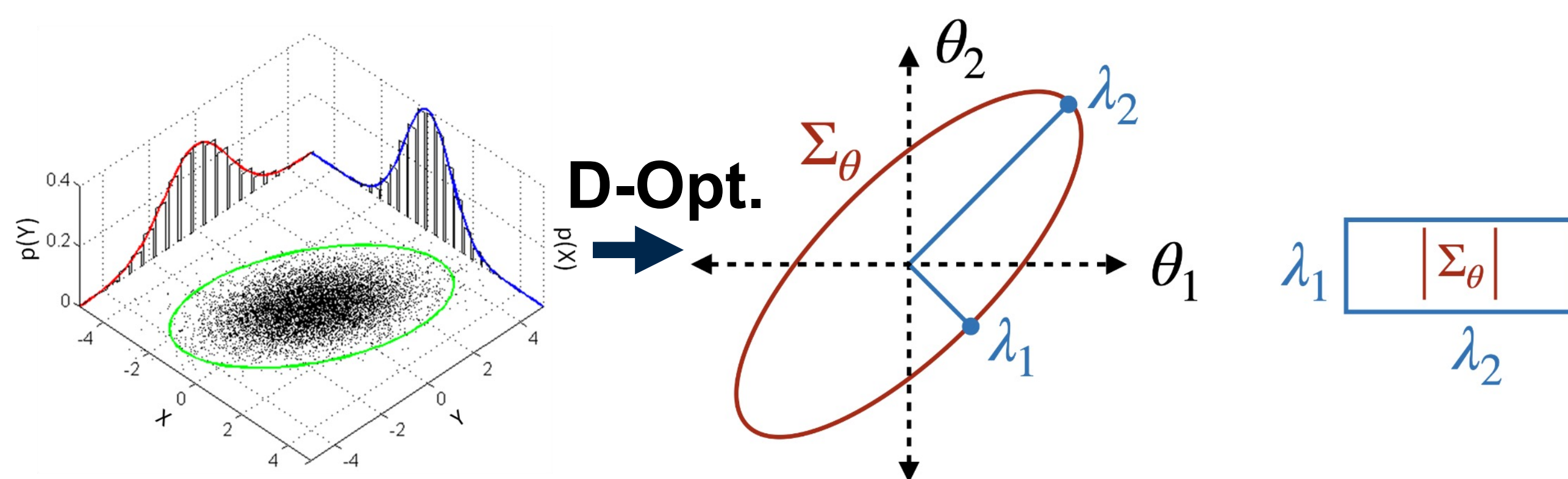
## P-Optimality (T.O.E.D.)

- Formulate information gain as a reduction of uncertainty.
- Quantified as a function of covariance matrix, depending on a scalar  $p$  [1].

Table 1. Properties of P-Optimality for different values of  $p$ . Note:  $\lambda_k$  represent the eigenvalues of the covariance matrix  $\Sigma_i$ .

|                     | T-optimality   | A-optimality   | D-optimality   | E-optimality                        |
|---------------------|--|--|--|-------------------------------------|
| $p$                 | 1  | -1   | 0  | $\mp\infty$                         |
| Equivalent Formulae | $\frac{1}{l}\text{tr}(\Sigma_i) = \frac{1}{l}\sum_k \lambda_k$ | $\left(\frac{1}{l}\text{tr}(H_i)\right)^{-1} = \left(\frac{1}{l}\sum_k \lambda_k^{-1}\right)^{-1}$ | $\sqrt[l]{ \Sigma_i } = \exp\left(\frac{1}{l}\sum_k \log \lambda_k\right)$ | $\min_{\lambda_k} \max_{\lambda_k}$ |
| Meaning             | Average Variance   | Harmonic Mean Variance   | Volume of covariance hyper-ellipsoid                                       | Single extreme eigenvalue           |

- Each has a unique geometric interpretation



## Efficient and flexible information quantification in 3D-GS for active exploration and keyframe selection.



(a) Uniform Sampling (b) FisherRF (c) Ours (d) Ground Truth

## Method

1. Construct Hessian matrix (right) using Gauss-Newton Approximation  $H(\theta) = J_\theta^T J_\theta$
2. Relate covariance to Hessian  $\Sigma \propto H(\theta)^{-1}$  (asymptotic relationship in MLE for exponential family)
3. Approximate as diagonal or block-diagonal due to size.
4. Measure reduction in covariance with P-Optimality.

|             |  |  |  |  |
|-------------|--|--|--|--|
| Ellipsoid 1 | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,1}} \frac{dc_i}{d\theta_{1,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,1}} \frac{dc_i}{d\theta_{1,2}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,1}} \frac{dc_i}{d\theta_{2,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,1}} \frac{dc_i}{d\theta_{2,2}}$ |
|             | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,2}} \frac{dc_i}{d\theta_{1,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,2}} \frac{dc_i}{d\theta_{1,2}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,2}} \frac{dc_i}{d\theta_{2,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{1,2}} \frac{dc_i}{d\theta_{2,2}}$ |
| Ellipsoid 2 | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,1}} \frac{dc_i}{d\theta_{1,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,1}} \frac{dc_i}{d\theta_{1,2}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,1}} \frac{dc_i}{d\theta_{2,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,1}} \frac{dc_i}{d\theta_{2,2}}$ |
|             | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,2}} \frac{dc_i}{d\theta_{1,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,2}} \frac{dc_i}{d\theta_{1,2}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,2}} \frac{dc_i}{d\theta_{2,1}}$ | $\sum_{i \in N} \frac{dc_i}{d\theta_{2,2}} \frac{dc_i}{d\theta_{2,2}}$ |

Hessian Matrix Approximation

## Results

### 1. Next Best View



| Method           | PSNR ( $\uparrow$ ) | SSIM ( $\uparrow$ ) | LPIPS ( $\downarrow$ ) |
|------------------|---------------------|---------------------|------------------------|
| Uniform Sampling | 23.32               | 0.885               | 0.101                  |
| FisherRF [2]     | 24.59               | 0.897               | 0.091                  |
| A-Opt. (Simple)  | 22.39               | 0.876               | 0.116                  |
| E-Opt. (Simple)  | 21.40               | 0.862               | 0.129                  |
| T-Opt. (Simple)  | 25.40               | 0.908               | 0.080                  |
| D-Opt. (Simple)  | <b>25.52</b>        | <b>0.909</b>        | <b>0.078</b>           |
| D-Opt. (Block)   | <u>25.41</u>        | <u>0.908</u>        | <u>0.078</u>           |

(a) Mip-Nerf360 [3] Data

| Method           | PSNR ( $\uparrow$ ) | SSIM ( $\uparrow$ ) | LPIPS ( $\downarrow$ ) |
|------------------|---------------------|---------------------|------------------------|
| Uniform Sampling | 17.29               | 0.508               | 0.432                  |
| FisherRF [2]     | 16.81               | 0.493               | 0.445                  |
| A-Opt. (Simple)  | 15.55               | 0.452               | 0.480                  |
| E-Opt. (Simple)  | 15.33               | 0.436               | 0.488                  |
| T-Opt. (Simple)  | 17.91               | 0.520               | 0.420                  |
| D-Opt. (Simple)  | <b>17.95</b>        | <b>0.535</b>        | <b>0.411</b>           |
| D-Opt. (Block)   | <b>18.15</b>        | <b>0.548</b>        | <b>0.401</b>           |

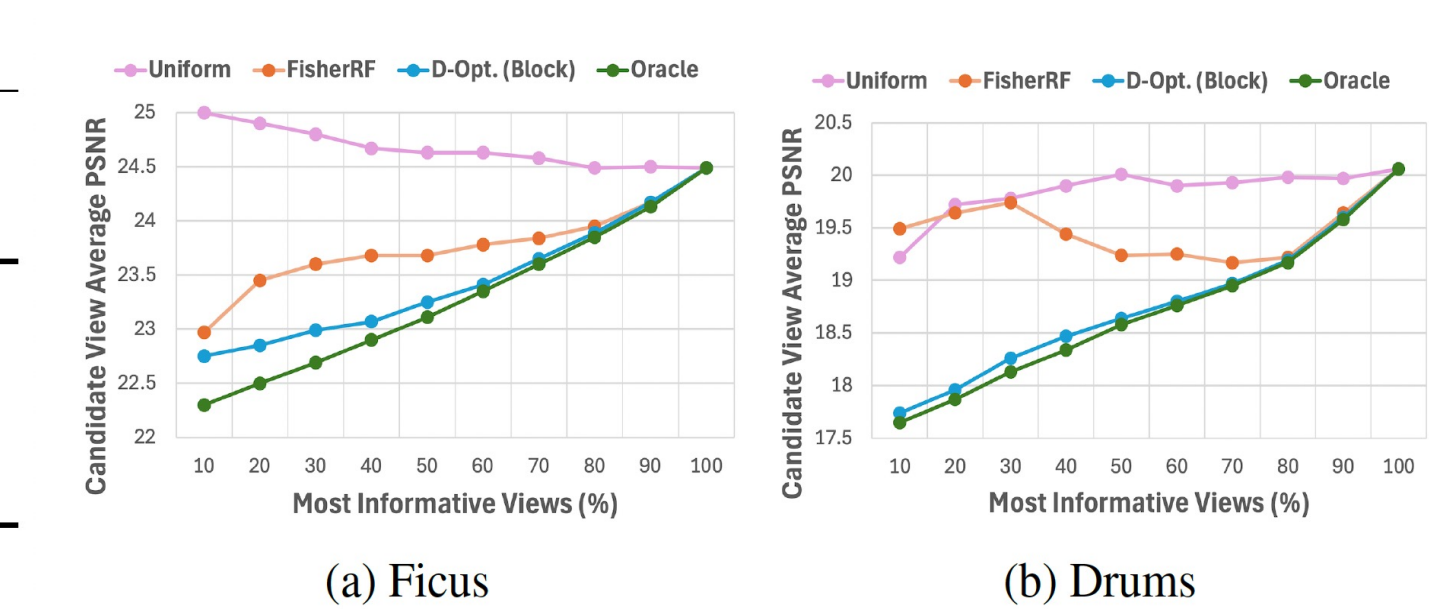
(b) Blender [4] Data

### 2. Keyframe Selection

| Method           | PSNR ( $\uparrow$ ) | SSIM ( $\uparrow$ ) | LPIPS ( $\downarrow$ ) |
|------------------|---------------------|---------------------|------------------------|
| Uniform Sampling | 23.47               | 0.888               | 0.109                  |
| FisherRF         | 18.37               | 0.829               | 0.184                  |
| A-Opt. (Simple)  | 17.05               | 0.811               | 0.226                  |
| E-Opt. (Simple)  | 16.55               | 0.786               | 0.255                  |
| T-Opt. (Simple)  | <b>24.90</b>        | <b>0.903</b>        | <b>0.096</b>           |
| D-Opt. (Simple)  | 24.26               | 0.899               | 0.101                  |
| D-Opt. (Block)   | <u>24.53</u>        | <u>0.902</u>        | <u>0.099</u>           |

T and D Opt. most effective

### 3. Uncertainty Correlation



Uncertainty correlated w/ image quality

## References

1. Jack Kiefer, "General equivalence theory for optimum designs (approximate theory)". *The Annals of Statistics*, 1974.
2. Jiang et al., "FisherRF: Active View Selection and Uncertainty Quantification for Radiance Fields using Fisher Information", *ECCV*, 2024.
3. Barron et al., "Mip-NeRF 360: Unbounded Anti-Aliased Neural Radiance Fields", *CVPR*, 2022.
4. Mildenhall et al., "NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis", *ICCV*, 2020.