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# Variational Graph-based Normal Integration

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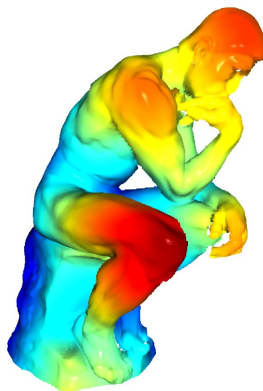
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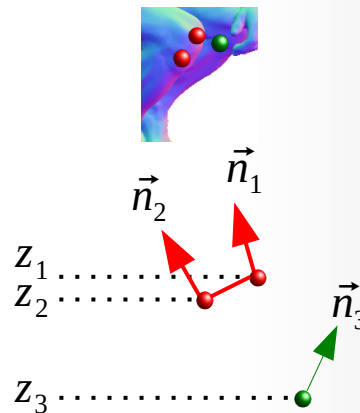
# Normal Integration



$$(x, y, \vec{n})$$



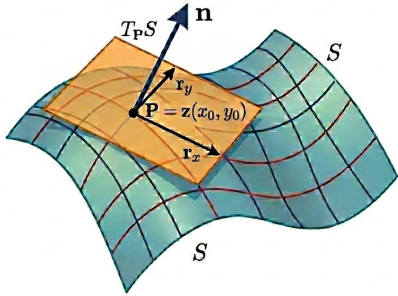
$$(x, y, z)$$



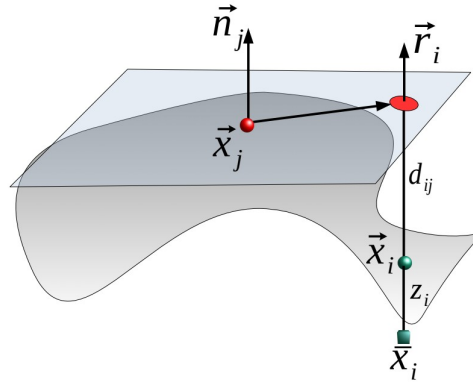
Challenge: Identifying and estimating depth discontinuities with per-point precision.

Objective: A graph-based numerical optimizer solves for  $z(x, y)$  in the presence of depth discontinuity.

# Surface as a Graph



Total Derivative defines the equation relating per-point normal and depth with 2 DoF.

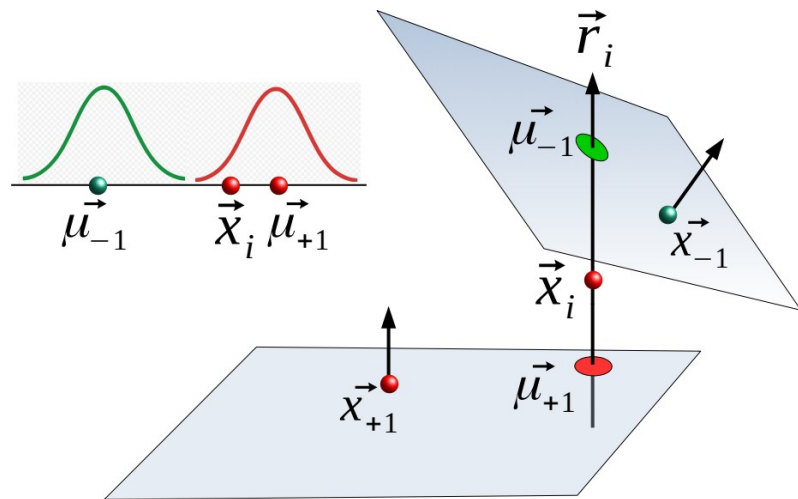


Point to Plane Distance quantifies the fitting error to the equation.

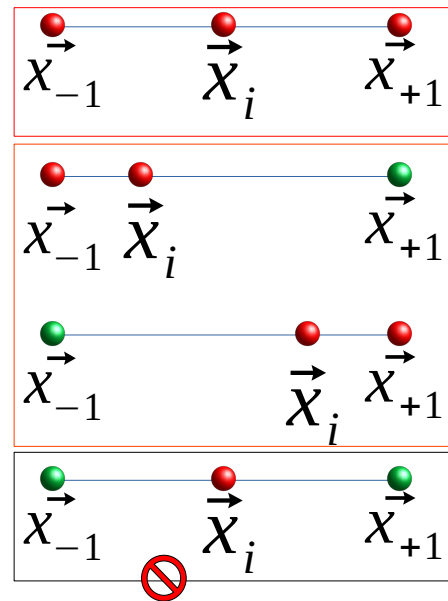
- Each point is a node in graph
- Point to Plane Distance pairs two neighboring nodes
- Small distance indicates differentiability
- Large distance implies depth discontinuity

$$\left(n_z + n_x \frac{u}{f^x} + n_y \frac{v}{f^y}\right) \Delta \ln z + n_x \frac{\Delta u}{f^x} + n_y \frac{\Delta v}{f^y} = 0$$

# Triplet Configuration

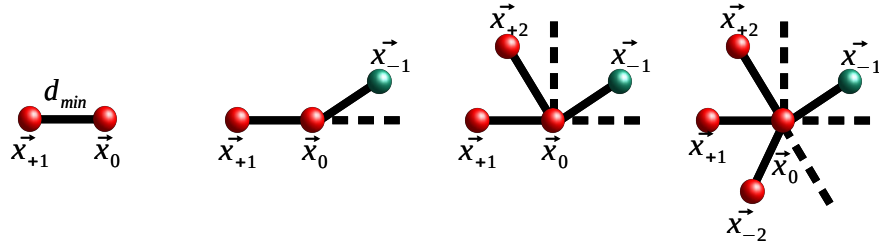


A triplet defines the per-point association of each graph node with two of its neighbors.



Each triplet is modeled by a two-component Gaussian mixture, ensuring one-sided connectivity.

# Shape Inference



$$w_{0,+1} = \sigma(d_{0,-1} - d_{0,+1})$$

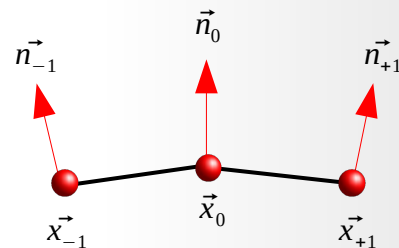
$$w_{0,-1} = \sigma(d_{0,+1} - d_{0,-1})$$

- At least two triplets are sampled for each node in each iteration.
- Each triplet is configured with largest angular separation
- The edge with minimum distance is always selected

# Depth Estimation

$$\begin{bmatrix} w_{0,+1} & 0 \\ 0 & w_{0,-1} \end{bmatrix} \begin{bmatrix} n_{z,+1} & -n_{z,+1} & 0 \\ 0 & n_{z,-1} & -n_{z,-1} \end{bmatrix} \begin{bmatrix} z_{+1} \\ z_0 \\ z_{-1} \end{bmatrix} = \begin{bmatrix} -n_{x,+1} \Delta x_{0,+1} - n_{y,+1} \Delta y_{0,+1} \\ -n_{x,-1} \Delta x_{0,-1} - n_{y,-1} \Delta y_{0,-1} \end{bmatrix}$$

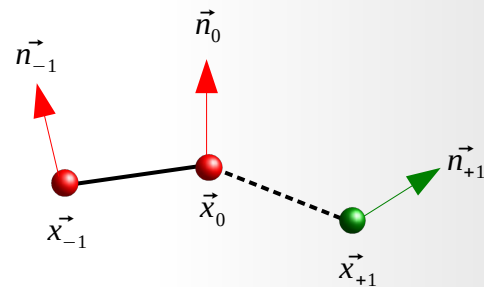
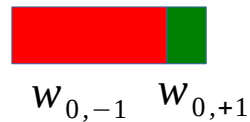
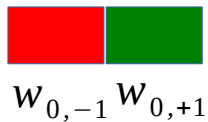
- Each triplet defines a system of two equations
- Both  $w$  and  $\vec{n}$  are inferred from current triplet configuration
- Depth vector  $\vec{z}$  is solved by conjugate gradients in a sparse Poisson system
- Solution minimizes  $w_{0,+1} d_{0,+1} + w_{0,-1} d_{0,-1}$



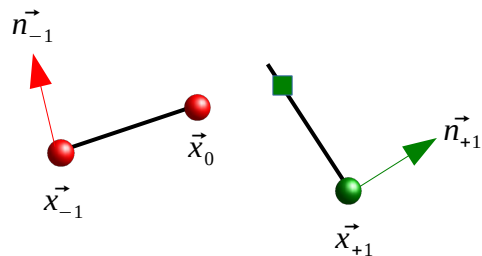
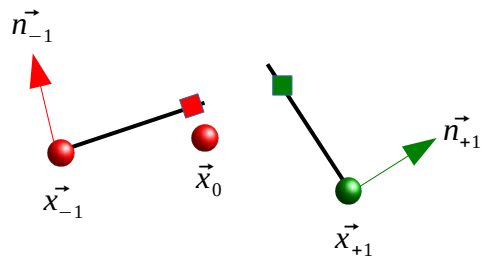
# Optimization

$$\min \sum_{i,j,k} w_{i,j} d_{i,j} + w_{i,k} d_{i,k}$$

- Shape inference



- Depth Estimation



# Experiment

## Orthographic

| Front     | bear        | buddha      | cow         | pot         | reading     | Side      | bear        | buddha      | cow         | pot        | reading     |
|-----------|-------------|-------------|-------------|-------------|-------------|-----------|-------------|-------------|-------------|------------|-------------|
| GNI(ours) | <b>0.60</b> | <b>1.91</b> | 5.62        | <b>0.59</b> | <b>2.42</b> | GNI(ours) | <b>1.24</b> | 4.04        | <b>0.36</b> | <b>7.2</b> | <b>2.87</b> |
| BiNI [7]  | 0.74        | 2.07        | <b>0.65</b> | 1.30        | 14.73       | BiNI [7]  | 1.72        | <b>3.81</b> | 9.84        | 11.51      | 5.06        |
| IPF [6]   | 24.29       | 8.23        | 2.50        | 4.07        | 33.59       | IPF [6]   | 29.29       | 10.79       | 38.66       | 17.35      | 10.67       |
| MD [15]   | 0.68        | 6.73        | 1.67        | 1.33        | 10.50       | MD [15]   | 13.46       | 20.99       | 42.44       | 19.84      | 31.23       |

## Effect of triplet sampling

| Front         | bear | buddha | cow   | pot  | reading |
|---------------|------|--------|-------|------|---------|
| GNI(w/ tri.)  | 1.26 | 6.05   | 18.72 | 1.29 | 5.08    |
| GNI(w/o tri.) | 1.91 | 7.71   | 35.70 | 1.73 | 10.14   |

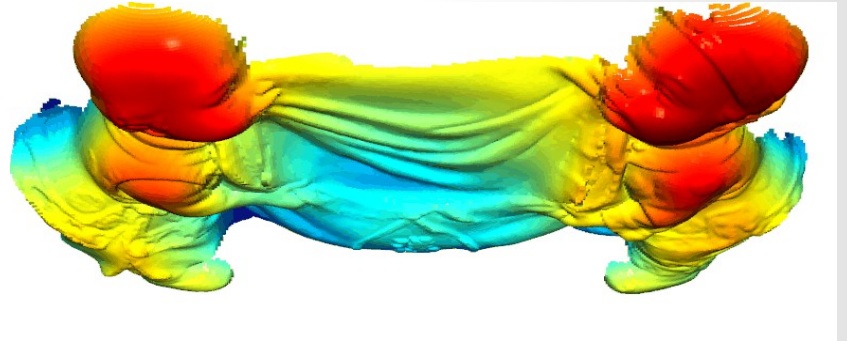
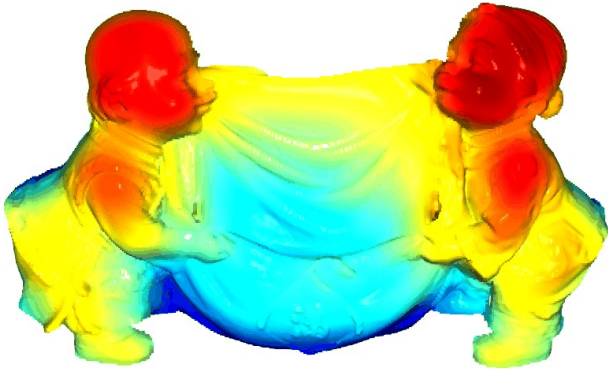
| Side         | bear  | buddha | cow  | pot   | reading |
|--------------|-------|--------|------|-------|---------|
| GNI(w/ tri.) | 5.99  | 9.65   | 1.15 | 14.44 | 17.62   |
| BS(w/o tri.) | 17.74 | 18.48  | 2.21 | 19.35 | 31.23   |

## Perspective

| Model      | bear | buddha | cat  | cow  | goblet | harvest | pot1 | pot2 | reading |
|------------|------|--------|------|------|--------|---------|------|------|---------|
| BiNI [7]   | 0.49 | 0.77   | 0.10 | 0.06 | 8.51   | 3.34    | 0.62 | 0.21 | 0.32    |
| GNI (ours) | 0.47 | 0.73   | 0.11 | 0.06 | 9.75   | 2.83    | 0.59 | 0.20 | 0.26    |



# Result



<https://github.com/ActiveVisionLab>