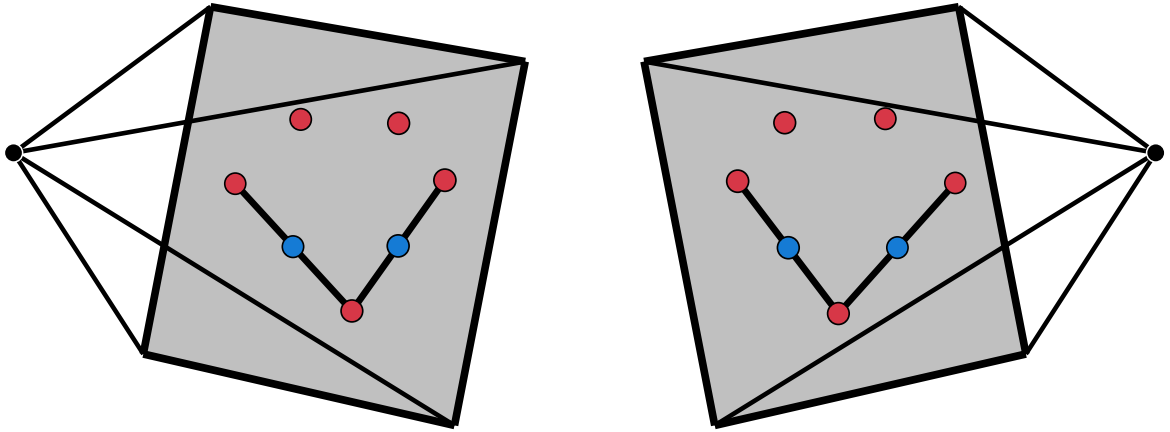

Linear Fundamental Matrix Estimation from 7 or 5 Points

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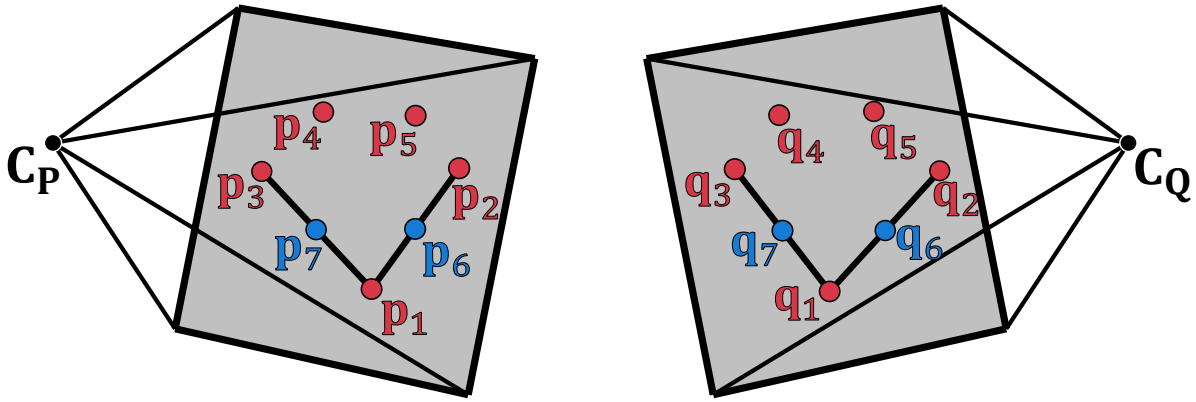
CVPR JUNE 3-7, 2026 DENVER COLORADO

University of Missouri

Main Contributions

Problem

Recover the relative geometry of two cameras by **estimating the fundamental matrix** from 2D point correspondences



V-Umlaut configuration with **5 independent** and **2 dependent points (real or virtual)**

Theoretical contributions

- **First linear solver** for the **minimal** fundamental matrix problem for the **V-Umlaut configuration**
- **Analysis of spurious solutions** computed by the standard 7-point algorithm under this configuration

Experimental contributions

- **Heuristic 5-point linear solver** using 2 virtual points to form a **V-Umlaut** point-line configuration from any 5 input point correspondences
- **PoseLib implementation as a drop-in replacement** in existing pipelines, with competitive RANSAC performance

Why Fundamental Matrix Solvers Matter

Fundamental matrix solvers are part of many core 3D vision pipelines.

Figure from "Building Rome in a Day" (Agarwal et al., 2011).

Improving these solvers benefits a wide range of applications.

- **Structure-from-Motion** *COLMAP, GLOMAP*
- **Monocular SLAM** *ORB-SLAM2 / 3*
- **Downstream Applications** *Novel-view synthesis with NeRF and Gaussian splatting*

F-matrix information appears in learned reconstruction models.

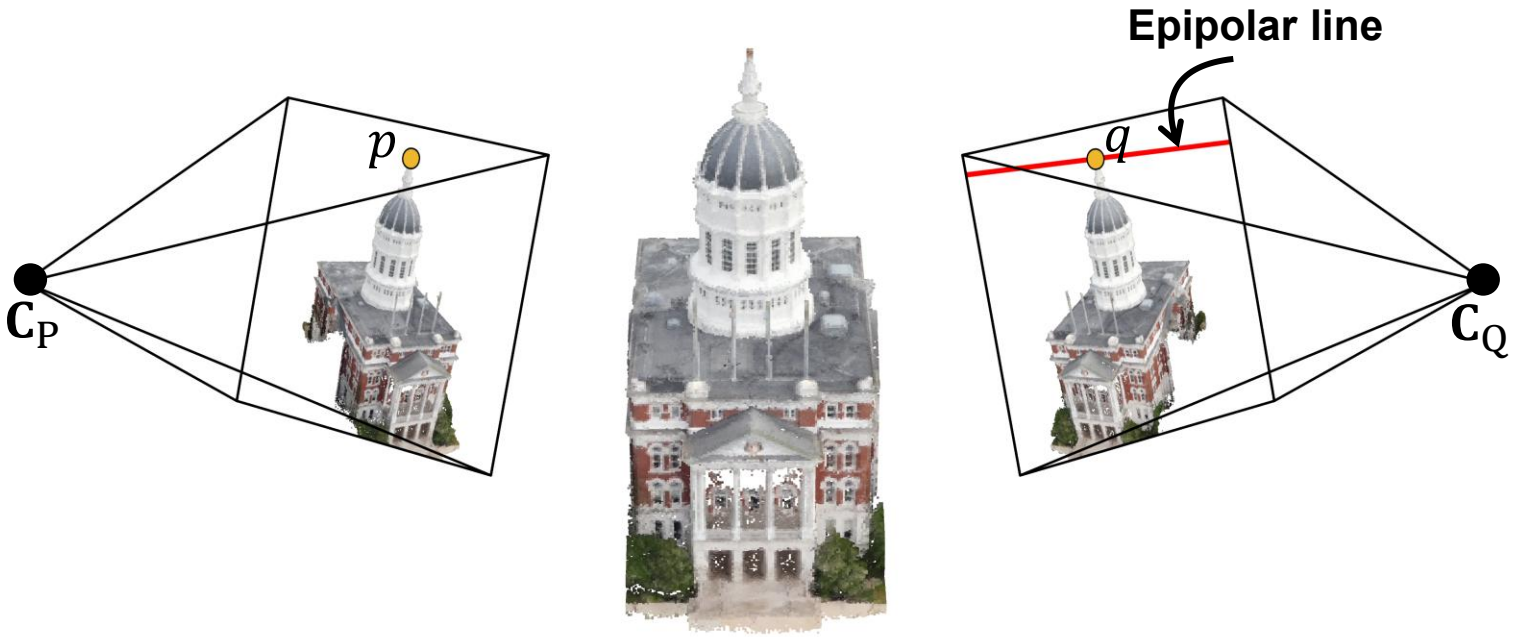
- **Feed-forward 3D reconstruction models** *DUSt3R, VGGT, Depth Anything 3*

J. Bratulić, S. Mittal, T. Brox, and C. Rupprecht, **On Geometric Understanding and Learned Priors in Feed-forward 3D Reconstruction Models**, *arXiv*, 2026.

Problem Formulation

Fundamental matrix $\mathbf{F}$

3 × 3 matrix of **rank 2**, defined up to scale (7 DoF)



Epipolar line: $\mathrm{l} = \mathbf{F}\,\mathbf{p}$

Constraints

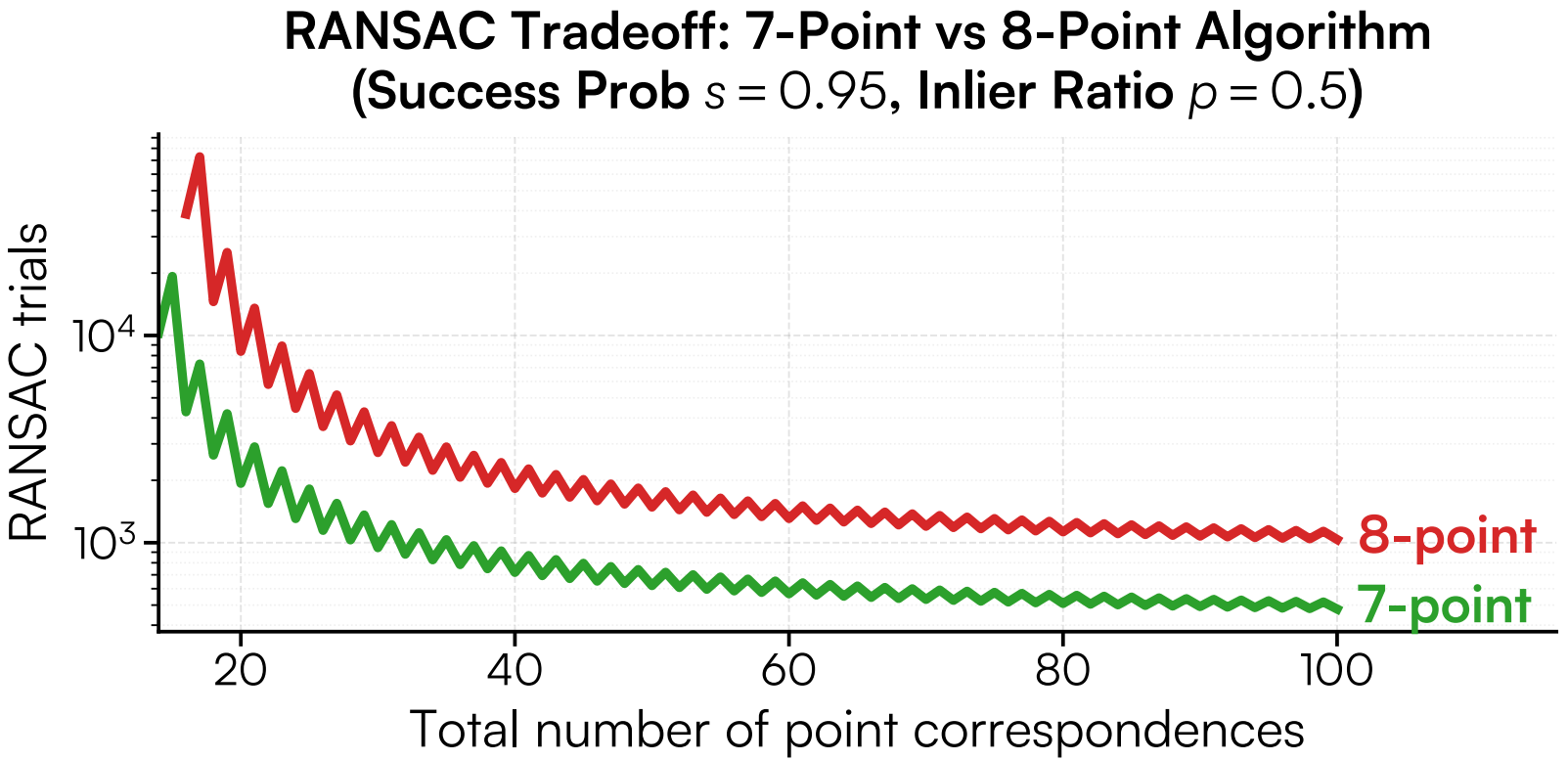
- Rank 2: $\det \mathbf{F} = 0$
- For each $\mathbf{p} \leftrightarrow \mathbf{q}$:
  Epipolar: $\mathbf{q}^\top \mathbf{F}\,\mathbf{p} = 0$

Solvers

V-Umlaut **(Ours)**	**Linear, Minimal** **5 independent** + **2 dependent** points
8-point	Linear, Non-Minimal
7-point	Nonlinear, Minimal Up to 3 solutions

Sample-Efficiency Advantage of Minimal Solvers

In RANSAC-based robust estimation, minimal solvers are the most sample-efficient estimators of $\mathbf{F}$.



V-Umlaut Solver — Per-View Projective Normalization

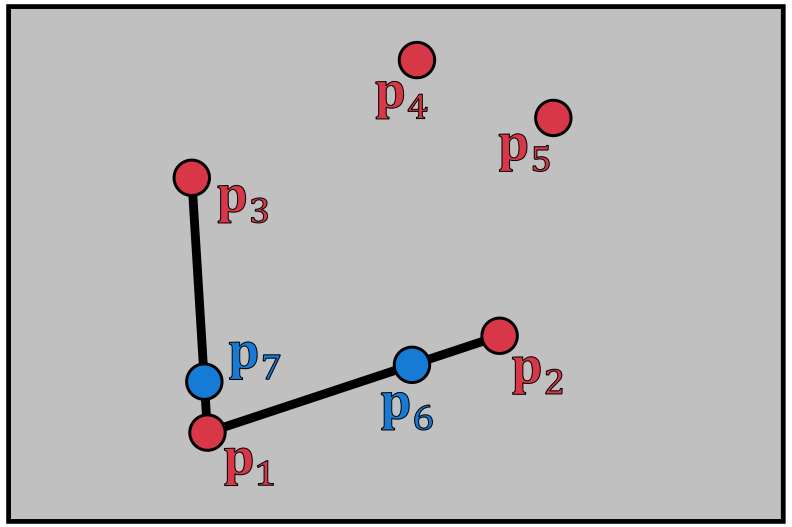


Image 1 (6600x4400 resolution)

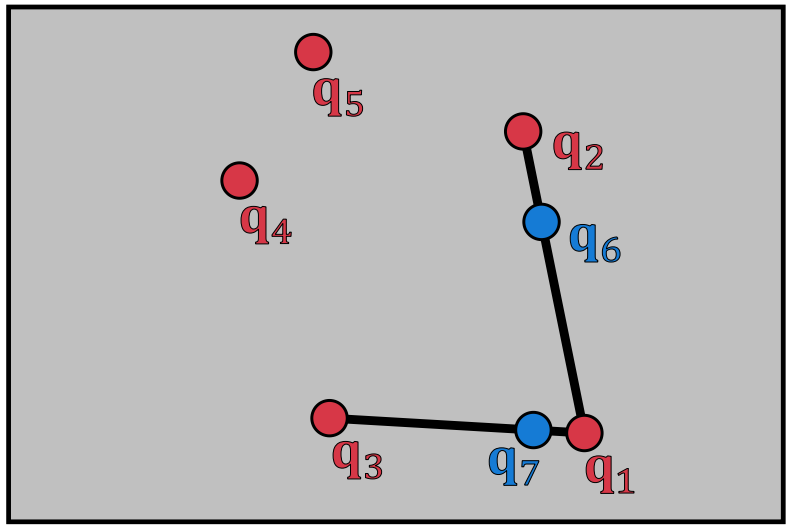


Image 2 (6600x4400 resolution)

$$\underset{\mathbf{p}_1}{\begin{bmatrix}1704\\3628\\1\end{bmatrix}} \quad \underset{\mathbf{p}_2}{\begin{bmatrix}4178\\2808\\1\end{bmatrix}} \quad \underset{\mathbf{p}_3}{\begin{bmatrix}1571\\1470\\1\end{bmatrix}} \quad \underset{\mathbf{p}_4}{\begin{bmatrix}3478\\473\\1\end{bmatrix}} \quad \underset{\mathbf{p}_5}{\begin{bmatrix}4633\\963\\1\end{bmatrix}} \quad \underset{\mathbf{p}_6}{\begin{bmatrix}3435.8\\3054\\1\end{bmatrix}} \quad \underset{\mathbf{p}_7}{\begin{bmatrix}1677.4\\3196.4\\1\end{bmatrix}}$$

Homography (H_1)

$$\begin{bmatrix}1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\end{bmatrix} \quad \begin{bmatrix}1\\1\\1\end{bmatrix} \quad \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \qquad s_{(1,1)} \quad s_{(1,2)}$$

(barycentric coordinates)

Special coordinate frame Z

$$\underset{\mathbf{q}_1}{\begin{bmatrix}4896\\3628\\1\end{bmatrix}} \quad \underset{\mathbf{q}_2}{\begin{bmatrix}4378\\1074\\1\end{bmatrix}} \quad \underset{\mathbf{q}_3}{\begin{bmatrix}2738\\3502\\1\end{bmatrix}} \quad \underset{\mathbf{q}_4}{\begin{bmatrix}1976\\1490\\1\end{bmatrix}} \quad \underset{\mathbf{q}_5}{\begin{bmatrix}2600\\401\\1\end{bmatrix}} \quad \underset{\mathbf{q}_6}{\begin{bmatrix}4533.1\\1840.0\\1\end{bmatrix}} \quad \underset{\mathbf{q}_7}{\begin{bmatrix}4464.3\\3602.8\\1\end{bmatrix}}$$

Homography (H_2)

$$\begin{bmatrix}1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\end{bmatrix} \quad \begin{bmatrix}1\\1\\1\end{bmatrix} \quad \begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} \qquad s_{(2,1)} \quad s_{(2,2)}$$

(barycentric coordinates)

Special coordinate frame W

V-Umlaut Solver — Spurious Solutions

After per-view normalization, $\mathbf{F}$ takes the form:

$$\mathbf{F} = \begin{bmatrix} 0 & f_{12} & f_{13} \\ f_{21} & 0 & f_{23} \\ f_{31} & 1 & 0 \end{bmatrix}$$

With the standard constraints:

$$\det(\mathbf{F}) = 0 \tag{1}$$

$$\mathbf{w}_i^{\top} \mathbf{F} \mathbf{z}_i = 0, \qquad i = 4, 5, 6, 7 \tag{2}$$

Returns **3 solutions** for $\mathbf{F}$, **2 of them spurious**

V-Umlaut Solver — Spurious Solutions

Spurious root 1:

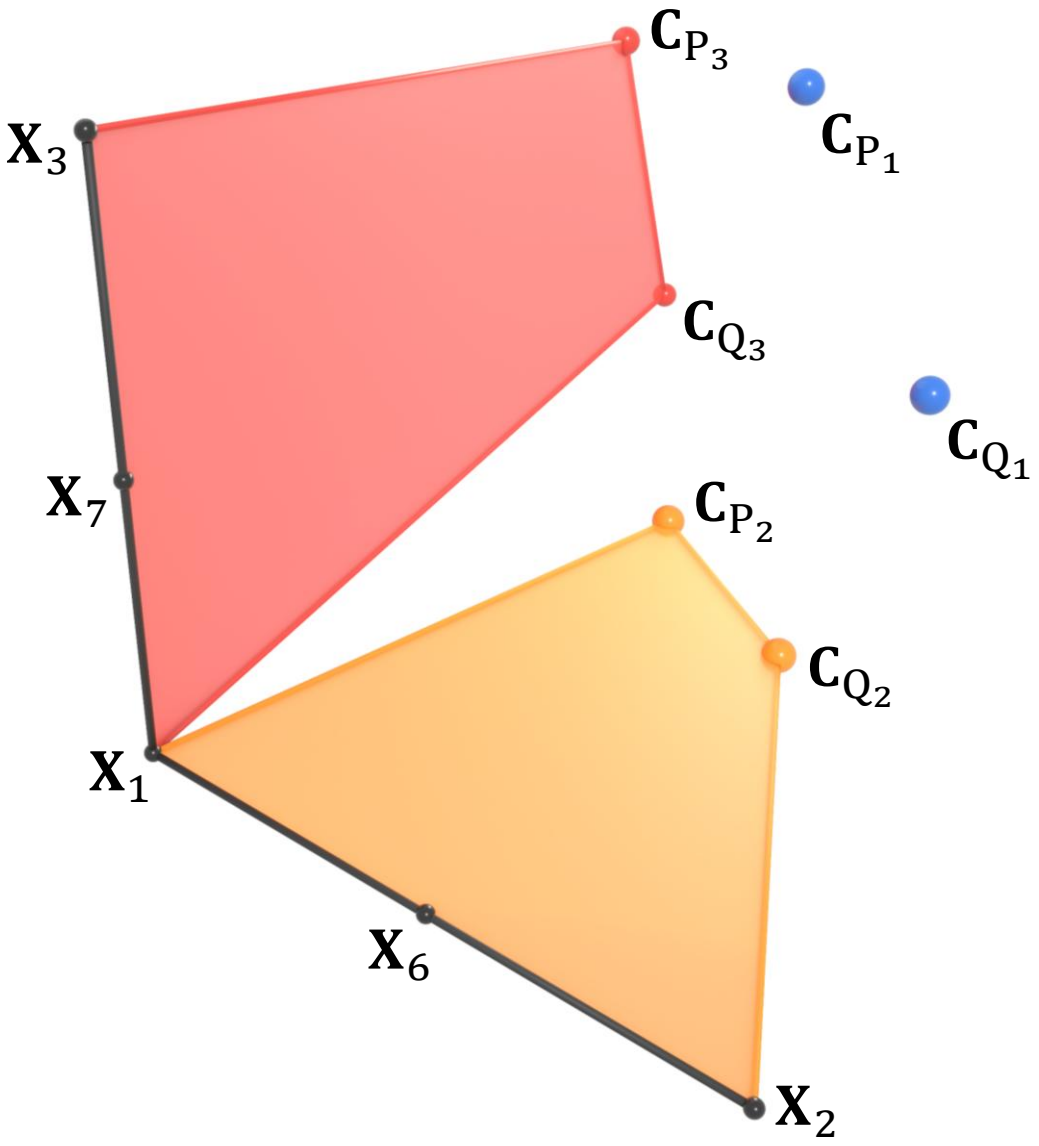
$\Rightarrow \mathbf{X}_1, \mathbf{X}_2, \mathbf{C}_{\mathrm{P}_2}, \mathbf{C}_{\mathrm{Q}_2}$ coplanar

Spurious root 2:

$\Rightarrow \mathbf{X}_1, \mathbf{X}_3, \mathbf{C}_{\mathrm{P}_3}, \mathbf{C}_{\mathrm{Q}_3}$ coplanar

Discarding spurious solutions

$\Rightarrow$ **unique solution for $\mathbf{F}$**



This explains a recent uniqueness result:

K. Kiehn, A. Ahlbäck, and K. Kohn, **PLMP – Point-Line Minimal Problems for Projective SfM,** *ICCV*, 2025.

V-Umlaut in Practice

V-Umlaut in Practice

Real-Data Experiments

Sample images of the Trevi Fountain scene from PhotoTourism

- **PhotoTourism** dataset is used with the experimental pipeline of Tzamos et al. [1, 2]
- 5000 image pairs sampled from each of 13 scenes, with **SuperPoint + SuperGlue** feature matches
- **PoseLib** [3] **LO-RANSAC** pipeline is used with input pixel correspondences normalized by the inverse camera intrinsic matrix ($\mathbf{K}^{-1}$)
- **COLMAP**-reconstructed camera poses used as pseudo ground-truth

Rotation error

$$\epsilon_R = \arccos\left(\frac{\mathrm{tr}(\mathbf{R}_{est}^{\top}\mathbf{R}_{gt}) - 1}{2}\right)$$

Translation error

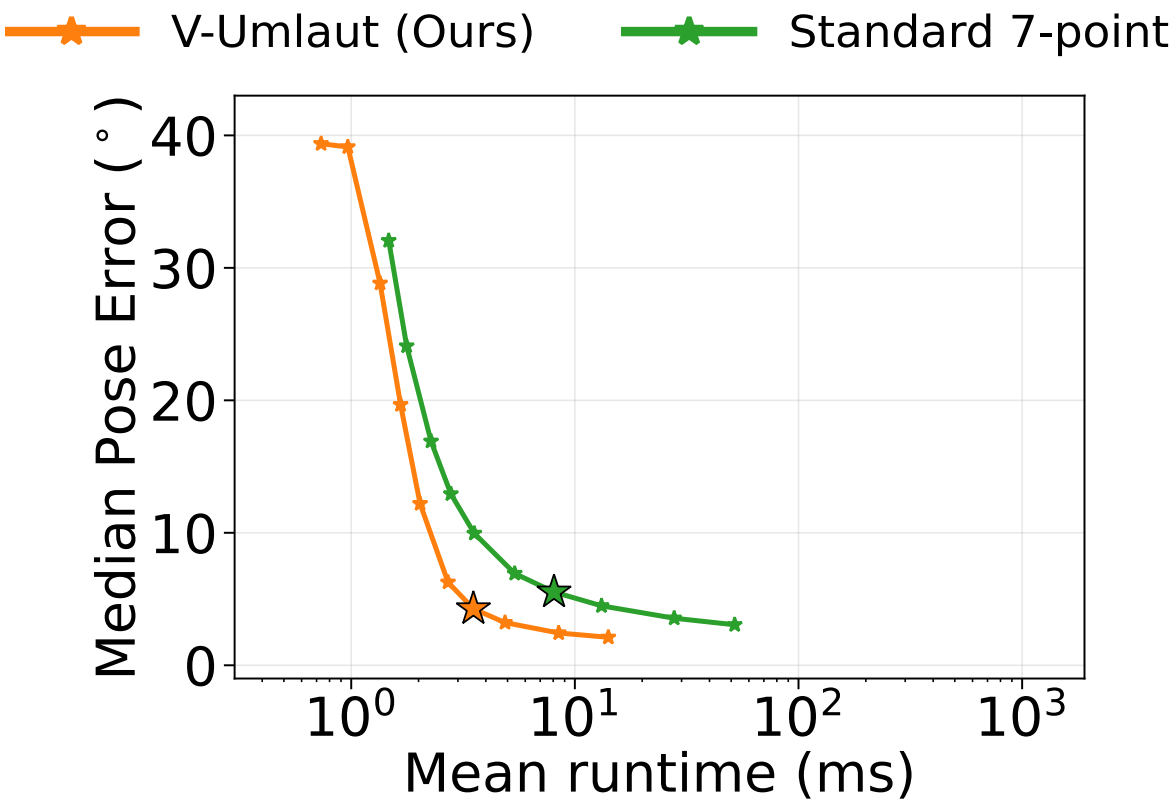
$$\epsilon_t = \arccos\left(\frac{\mathbf{t}_{est} \cdot \mathbf{t}_{gt}}{\|\mathbf{t}_{est}\| \, \|\mathbf{t}_{gt}\|}\right)$$

Pose error

$$\epsilon_{\text{pose}} = \max(\epsilon_R, \epsilon_t)$$

[1] Y. Jin et al., **Image Matching across Wide Baselines: From Paper to Practice**, *IJCV*, 2020.
[2] C. Tzamos et al., **Practical Solutions to the Relative Pose of Three Calibrated Cameras**, *CVPR*, 2025.
[3] V. Larsson and contributors, **PoseLib — Minimal Solvers for Camera Pose Estimation**, 2020. github.com/PoseLib/PoseLib

PhotoTourism Results



Runtime vs. pose accuracy results on **all 13 scenes from PhotoTourism.** All methods use the PoseLib LO-RANSAC implementation, with the number of RANSAC iterations in the set **{10, 20, 50, 100, 200, 500, 1000, 2000, 5000, 10000}** and a 1-pixel epipolar threshold.

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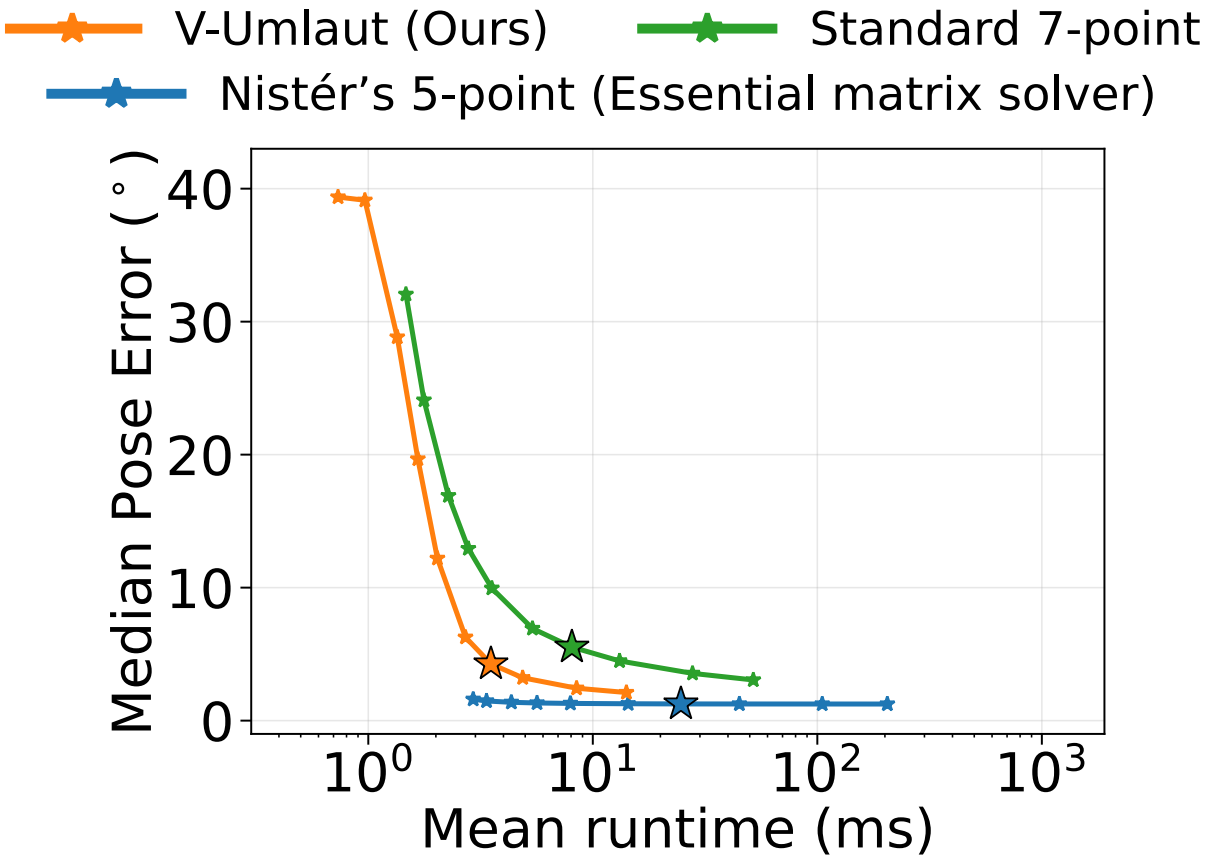
PhotoTourism Results



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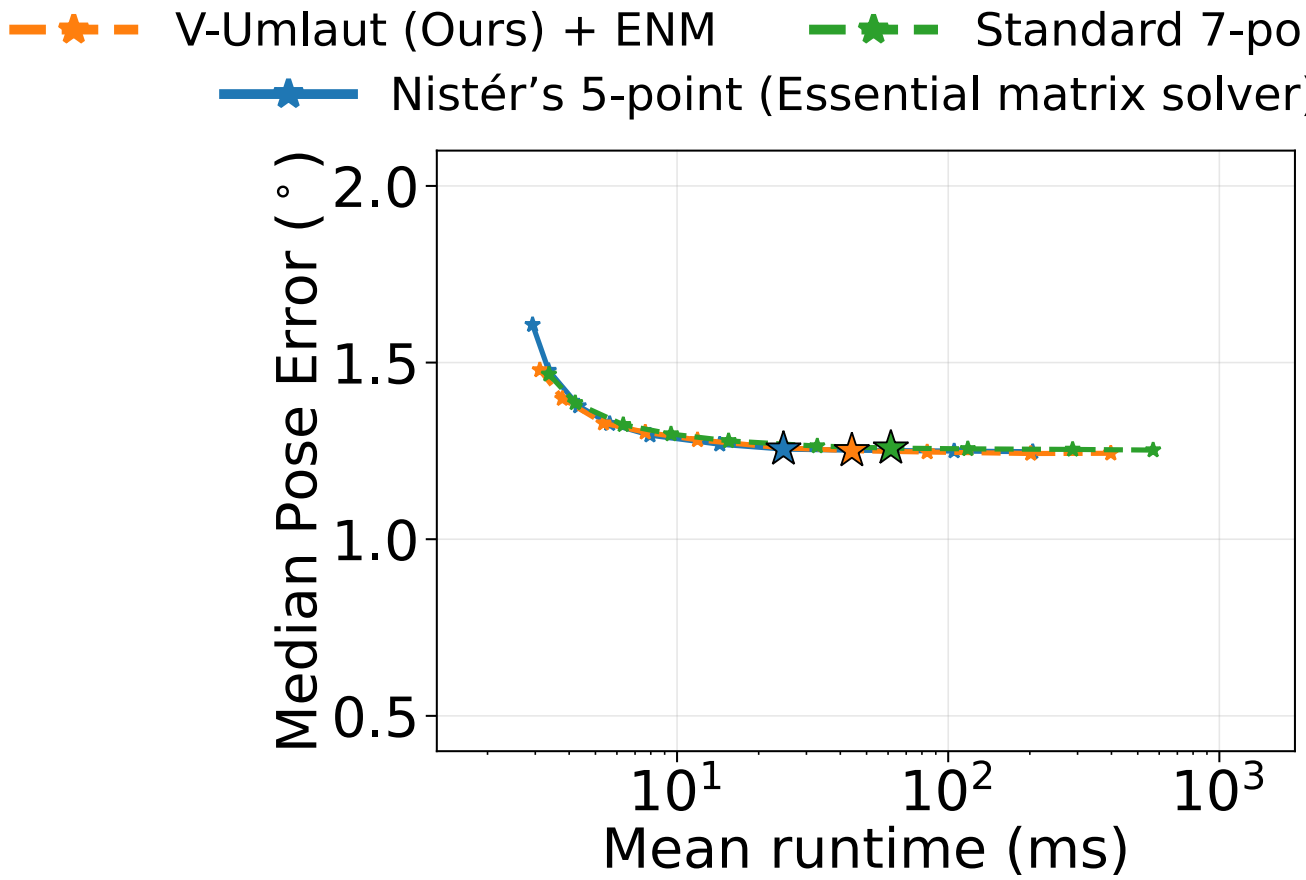
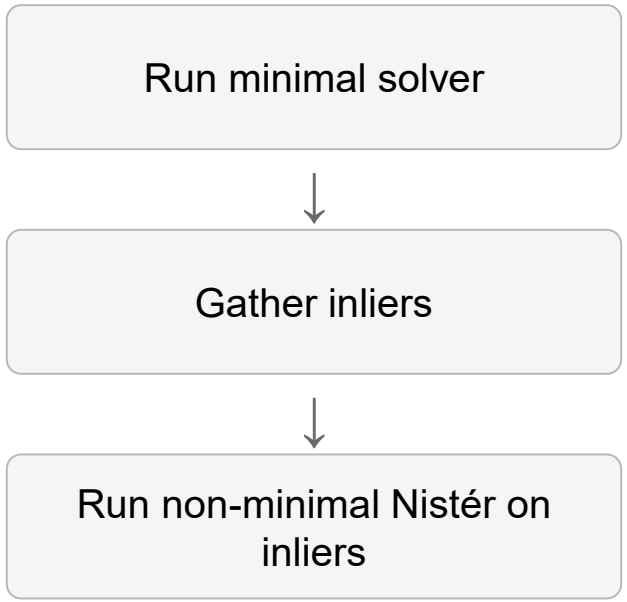
[3] V. Larsson and contributors, **PoseLib — Minimal Solvers for Camera Pose Estimation**, 2020. github.com/PoseLib/PoseLib
[4] D. Nistér, **An Efficient Solution to the Five-Point Relative Pose Problem**, IEEE TPAMI, 2004.

PhotoTourism Results



Early Non-Minimal Refitting (ENM)



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[3] V. Larsson and contributors, **PoseLib — Minimal Solvers for Camera Pose Estimation**, 2020. github.com/PoseLib/PoseLib
[4] D. Nistér, **An Efficient Solution to the Five-Point Relative Pose Problem**, IEEE TPAMI, 2004.

Conclusion

- V-Umlaut addresses a classical two-view geometry problem with a new linear minimal fundamental matrix solver, delivering competitive results against state-of-the-art in a RANSAC pipeline.

- We provide a V-Umlaut implementation integrated into PoseLib that drops into existing pipelines as a 5-point heuristic solver using 2 virtual midpoints.

v-umlaut.github.io